AD-A085 327 AIR FORCE AERO PROPULSION LAB WRIGHT-PATTERSON AFB OH F/G 20/4
CALCULATION TECHNIQUES FOR INVISCID TWO-DIMENSIONAL SUPERSONIC --ETC(U)
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AFAPL-TR-79-2023

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFAPL-TR-79-2023	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) CALCULATION TECHNIQUES FOR INVISCID TWO-DIMENSIONAL SUPERSONIC AIRFLOW		5. TYPE OF REPORT & PERIOD COVERED Final Report Sep 77 - Dec 78
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) M. Brian Bergsten		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Aero Propulsion Laboratory (RJT) AF Wright Aeronautical Laboratories Wright-Patterson Air Force Base, Ohio 45433		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Project 3012, Task 301211 Work Unit 30121101
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Aero Propulsion Laboratory (RJ) AF Wright Aeronautical Laboratories Wright-Patterson Air Force Base, Ohio 45433		12. REPORT DATE September 1979
		13. NUMBER OF PAGES 136
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Supersonic aerodynamics
Inviscid airflow
Two-dimensional airflow
Supersonic inlets
Supersonic wings

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Techniques for calculating the change in properties of inviscid supersonic airflow passing over or through various two-dimensional bodies are discussed in this report. Low supersonic (M<4) airflows and high supersonic (M>4) airflows are considered separately. Flow phenomena considered include normal shock waves, oblique shock waves, insentropic expansions, and conical shock waves for cones at zero degrees angle of attack. The adjustments required for swept wings are also discussed. Printouts of computer subroutines designed to carry out the described calculation procedures are included in an appendix to the report.

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Item 20 Continued:

Examples describing the use of the calculation techniques and computer subroutines for a swept wing, and for two typical ramjet inlets are included. A final appendix includes charts that can be used as calculation aids for airflows at Mach numbers up to 4.

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CALCULATION TECHNIQUES FOR INVISCID TWO-DIMENSIONAL SUPERSONIC AIRFLOW

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September 1979

TECHNICAL REPORT AFAPL-TR-79-2023

Final Report for Period September 1977 - December 1978

AIR FORCE AERO PROPULSION LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

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LIST OF SYMBOLS

a	Speed of sound - ft/sec
A	Area - ft^2
c	Maximum velocity attainable by expansion to absolute zero temperature - ft/sec
c_p	Specific heat at constant pressure - Btu/lb^{o}R
e	Base of the system of natural logarithms (c=2.71828...)
F	Force along wing centerline - lbs
g	Gravitational constant (g=32.174 ft/sec)
h	Static enthalpy - Btu/lb
h_{surf}	Surface static enthalpy - Btu/lb
h_t	Total enthalpy - Btu/lb
J	Conversion factor (J=778.15 ftlb/Btu)
l	Wing cord length - ft
M	Mach number
M_{2w}	Mach number downstream of shock wave on a two-dimensional deflection
M_{∞}	Freestream Mach number
$M_{\infty e}$	Equivalent freestream Mach number
$M_{\infty n}$	Component of freestream Mach number normal to shock wave
$M_{\infty t}$	Component of freestream Mach number tangential to shock wave
N	Wing normal force - lbs
P	Static pressure - lb/ft^2
P_t	Total pressure - lb/ft^2
r	Radial distance - ft
R	Gas constant for air at standard conditions (R=53.3 ftlb/lb^{o}R)
s	Entropy - Btu/lb^{o}R
t	Wing thickness - ft

LIST OF SYMBOLS (CONCLUDED)

T	Static temperature - ^{o}R
T_t	Total temperature - ^{o}R
u	Velocity in radial direction - ft/sec
u_{surf}	Surface velocity - ft/sec
v	Velocity in direction perpendicular to cone surface - ft/sec
V	Velocity - ft/sec
w	Velocity in ϕ direction downstream of conical shock wave - ft/sec
y	Distance defined by vector diagram on page 34
α	Angle of attack - degrees
α_e	Equivalent angle of attack - degrees
γ	Ratio of specific heats
γ_I	Isentropic exponent
δ	Deflection angle - degrees
Δ	Indicates incremental change in following parameter
θ	Shock wave angles - degrees
θ_{cowl}	Cowl angle - degrees
θ_r	Ray angle - degrees
λ	Wing sweep angle - degrees
μ	Mach angle - degrees
ν	Prandtl-Meyer angle - degrees
,	Density based on static properties - lb/ft^3
ρ_t	Density based on total properties - lb/ft^3
σ	Wing wedge angle - degrees
σ_e	Equivalent wing wedge angle - degrees
ϕ	Flow direction downstream of conical shock wave - degrees

SECTION I

INTRODUCTION

The calculation of flow parameters in a supersonic airstream is accomplished by application of the well-known conservation equations of fluid mechanics. These equations basically state that for a steadily flowing (time independent) system, the mass, momentum, and energy leaving a control volume must be equal to the mass, momentum, and energy entering the system, assuming that there is no change in these quantities within the control volume. The application of the conservation equations to the various phenomena that can occur in a supersonic airstream is quite straightforward, and for low supersonic Mach numbers is well documented. For higher supersonic Mach numbers where the ratio of specific heats (γ) cannot be assumed constant, the documentation is not quite as complete.

The purpose of this report is to present in a concise manner the calculation techniques for both low and high supersonic Mach number airflows. The flow phenomena addressed include normal shock waves, oblique shock waves, and isentropic expansions. While all the techniques included are generally applicable only to two-dimensional flow, the special case of axisymmetric flow over a cone at zero angle of attack is also addressed. In addition, oblique shocks resulting from flow over swept wings are considered.

The calculation techniques for low supersonic Mach numbers are presented in Section II. These techniques employ constant γ equations and are quite straightforward. Separate subsections under Section II cover normal shock waves, two-dimensional oblique shock waves, isentropic expansions, and oblique shock waves for cones at zero degrees angle of attack. Included in each subsection is a discussion of a computer subroutine that has been written to be used as a computation aid. Section II.1 covers two-dimensional oblique shock waves. Included in Section II.1 is the derivation of a closed form solution for determining shock wave angle, given the upstream Mach number and the deflection angle. An additional feature of Section II.1 is the inclusion

of charts that can be used in determining properties downstream of an oblique shock. These charts have been drawn to include upstream Mach numbers as high as Mach 4. Section II.4 (on shock waves emanating from the apex of a zero degree angle of attack cone) also includes charts useful for determining conditions downstream of the shock wave. Example cases illustrating the use of the charts are included.

Calculation techniques for high Mach number supersonic flows are presented in Section III. As in Section II, separate subsections are included for each of the various flow phenomena. For the high supersonic Mach number solutions, precise closed form solutions are not available, and the calculation techniques are all iterative solutions employing the conservation equations and an approximate model for equilibrium air devised by Hansen and Hodge as reported in NASA TN D-352. A versatile computer subroutine based on this model has been developed and is described in Section III. Each of the computer subroutines written for the various flow phenomena employs this subroutine for equilibrium air calculations. Because of the number of variables involved, a set of charts such as presented in Section II is not possible for calculation of the high Mach number flow parameters. The computer subroutines presented in Section III are the most expedient means for obtaining the required information.

Computer programs employing the subroutines are included in Section IV for several hypothetical cases. Solutions are also worked out for each of the cases, using the charts described in Section II, in order to illustrate the magnitude of the error resulting from the use of the charts at various Mach numbers. All of the examples discussed in Section IV are actually very idealized cases. No flow process is truely two-dimensional; since interactions with sideplates or edges will occur in flow over any surface of non-infinite width. Additionally, the effects of the viscosity of the air have not been considered in the calculation techniques. Viscous interactions and three-dimensional effects are beyond the scope and purpose of this report. Since these effects that occur in any real situation, are not included, the results will not necessarily be in close agreement with data obtained from actual test

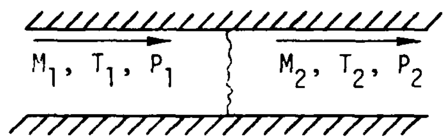
hardware. To predict data as obtained from test hardware, one must employ empirical methods, derived from data obtained at similar conditions, in conjunction with the calculation techniques reported herein. This also is beyond the scope and purpose of this report. Therefore, no comparisons with test data are included in the report.

SECTION II

TECHNIQUES FOR LOW MACH NUMBER (M<4) SUPERSONIC AIR FLOWS

The well known calculation techniques described in this section are included primarily for completeness. For each of the flow phenomena considered (normal shock, oblique shock, isentropic expansion, and conical shock), a computer subroutine was written to perform the required calculations. These subroutines are available for use in a computer program that can easily be written to handle any two dimensional flow situation. They were also employed to calculate the data plotted on the charts associated with each of the calculation techniques.

1. NORMAL SHOCK WAVE



For flow calculations involving a normal shock, the Mach number, static pressure, and static temperature upstream of the shock wave are generally known, and it is desired to find the value of those parameters downstream. Also of interest is the entropy increase or total pressure ratio across the shock. Equations listed in NACA Report 1135 can be used to calculate these quantities. The derivation of the equations is trivial and will not be repeated here. The equations to be used are

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1} \qquad (1)$$

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - (\gamma-1)]\,[(\gamma-1) M_1^2 + 2]}{(\gamma+1)^2 M_1^2} \qquad (2)$$

$$M_2 = \sqrt{\frac{(\gamma-1) M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)}} \qquad (3)$$

$$\frac{P_{t_2}}{P_{t_1}} = \left[\frac{(\gamma+1)\; M_1^2}{(\gamma-1)\; M_1^2 + 2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}} \tag{4}$$

For convenience, the normal shock subroutine also includes the calculation of total pressure and temperature upstream and downstream of the shock wave. These parameters are found by applying the following equations:

$$\frac{T_t}{T} = 1 + \frac{\gamma-1}{2} M^2 \tag{5}$$

$$\frac{P_t}{P} = \left(1 + \frac{\gamma-1}{2} \; M^2\right)^{\frac{\gamma}{\gamma-1}} \tag{6}$$

Using Equations 1 through 6, the desired quantities may be easily calculated using non-iterative methods. The calling statement for the computer subroutine is

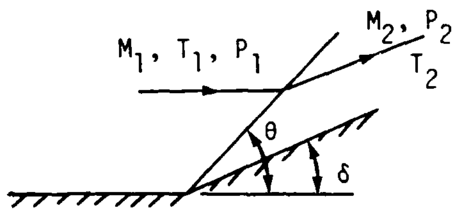
CALL NORM (GAM, T1, P1, M1, T2, P2, M2, PR, TT, PT1)

The first four items in the sequence are inputs. The remaining items are outputs, and for the most part are self explanatory. The units of the output quantities will be the same as the units of the corresponding input quantities. The quantity PR is the total pressure ratio (P_{t_2}/P_{t_1}). TT is the total temperature, and PTI is the upstream total pressure. The analyses herein are based on the assumption that the system being considered is adiabatic; thus, the total temperature upstream and downstream of the shock is the same. The entropy change across the shock is related to the total pressure ratio by the equation

$$\frac{\Delta S}{R} = -\ell n \left(\frac{P_{t_2}}{P_{t_1}}\right) \tag{7}$$

Subroutine NORM (Table A-1) was used to calculate the data presented in Figure B-1. Figure B-1 can be used in performing hand calculations for systems containing a normal shock. Data are plotted in Figure B-1 for Mach numbers up to 4. The method described in Section III should be used for Mach numbers greater than about 4 and for cases in which the flow upstream of the shock wave has an elevated temperature.

2. OBLIQUE SHOCK WAVE-PLANAR



A supersonic 2-D airflow encountering a positive deflection will experience an oblique shock wave. Generally the conditions upstream of the shock wave and the flow deflection angle are known, and it is desired to determine the conditions downstream and the angle of the shock wave. The derivation of the oblique shock equations is also well documented and will not be repeated here. The equations listed in NACA 1135 (Reference 2) for oblique shock waves will be used in the computer program. The equations used are

$$\frac{P_2}{P_1} = \frac{2\,\gamma\, M_1^2 \sin^2\theta - (\gamma-1)}{\gamma+1} \tag{8}$$

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 \sin^2\theta-(\gamma-1)]\,[(\gamma-1)\, M_1^2 \sin^2\theta+2]}{(\gamma+1)^2\, M_1^2 \sin^2\theta} \tag{9}$$

$$M_2 = \sqrt{\frac{(\gamma+1)^2\, M_1^4 \sin^2\theta-4(M_1^2 \sin^2\theta-1)(\gamma M_1^2 \sin^2\theta+1)}{[2\gamma M_1^2 \sin^2\theta-(\gamma-1)][(\gamma-1)\, M_1^2 \sin^2\theta+2]}} \tag{10}$$

$$\frac{P_{t_2}}{P_{t_1}} = \left[\frac{(\gamma+1)\, M_1^2 \sin^2\theta}{(\gamma-1)\, M_1^2 \sin^2\theta+2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M_1^2 \sin^2\theta-(\gamma-1)}\right]^{\frac{1}{\gamma-1}} \tag{11}$$

Equations 8 through 11 can be easily solved if the shock wave angle is known. However, as stated previously, the deflection angle is generally the known parameter, and shock wave angle is one of the parameters to be determined. An equation relating the deflection angle and the shock wave angle is

$$\cot\delta = \tan\theta \left[\frac{(\gamma+1)\, M_1^2}{2(M_1^2 \sin^2 \theta - 1)} - 1 \right] \tag{12}$$

Examination of Equation 12 will show that the use of this equation would require an iterative solution. To avoid the iterative solution another equation from NACA 1135 (Reference 2) is employed:

$$\sin^6 \theta + p \sin^4 \theta + q \sin^2 \theta + r = 0 \tag{13}$$

where

$$p = -\frac{M_1^2 + 2}{M_1^2} - \gamma \sin^2 \delta \tag{14}$$

$$q = \frac{2M_1^2 + 1}{M_1^4} + \left[\frac{(\gamma+1)^2}{4} + \frac{\gamma-1}{M_1^2} \right] \sin^2 \delta \tag{15}$$

$$m = -\frac{\cos^2\delta}{M_1^4} \tag{16}$$

Solving Equation 13 explicitly for θ is quite complicated, so the solution will be detailed below.

If we let: $$y = \sin^2\theta \tag{17}$$

Then: $$y^3 + py^2 + qy + m = 0 \tag{18}$$

If we let: $$y = (x - \frac{p}{3}) \tag{19}$$

Then: $$x^3 + b_1 x + b_2 = 0 \tag{20}$$

Where: $$b_1 = 1/3\ (3q-p^2) \tag{21}$$

$$b_2 = 1/27\ (2p^3-9pq + 27m) \tag{22}$$

The solutions to Equation 20 are

$$x_k = 2\sqrt{-\frac{b_1}{3}}\ \cos\left(\frac{\phi}{3} + 120°k\right),\ k = 0,\ 1,\ 2 \tag{23}$$

Where: $$\cos\phi = \mp\sqrt{-27b_2^2/4b_1^3} \tag{24}$$

In Equation 24 the upper sign is used if b_2 is positive, and the lower sign is used if b_2 is negative.

The strong shock solution is found from Equation 23 with k=0. The weak shock solution, which is the one generally of interest, is obtained by setting k=2 in Equation 24. Using this value of x_k in Equation 19 and then solving for θ in Equation 17 provides the shock angle of interest. The solution obtained with k=1 is of no physical significance. Once the shock angle has been obtained, it is a simple matter to determine the other parameters of interest by applying Equations 8 through 10. The calling statement for the oblique shock computer subroutine is

```
CALL CGSK (DELT, VM1, T1, P1, GAM, SQ, DELTM, THETA, VM2, T2, P2)
```

The first six items in this sequence are inputs. DELT is the deflection angle, VM1 is the Mach number upstream of the deflection, and T1 and P1 are the temperature and pressure upstream of the deflection. GAM is the value of specific heat ratio to be used for the calculations. If a strong shock solution is desired, SQ is set equal to one. If SQ is set equal to zero, the weak shock solution is calculated. DELTM is the maximum deflection angle for which an attached oblique shock can exist for the input value of M1. If DELTM is less than the input value of DELT, no solution exists, and all of the output parameters will be set equal to zero. THETA is the shock angle calculated, and VM2, T2, and P2 are the Mach number, temperature and pressure downstream of the shock.

The units of temperature and pressure will be the same as those of the inputs. All angles are input and output in degrees. Neither the total pressure ratio nor the entropy increase are directly calculated in this subroutine. However, with the static pressure and temperature known on both sides of the shock wave, the entropy increase is easily determined by use of the following equation:

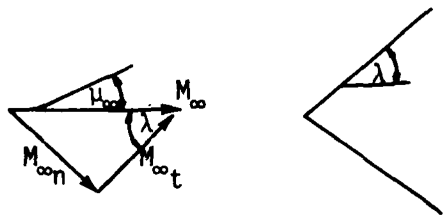
$$\frac{\Delta S}{R} = \frac{\gamma}{\gamma - 1} \ \ell n \frac{T_2}{T_1} - \ell n \frac{P_2}{P_1} \tag{25}$$

Total pressure ratio is then easily determined by application of the inverse of Equation 7:

$$\frac{P_{t_2}}{P_{t_1}} = e^{-\frac{\Delta S}{R}} \tag{26}$$

Subroutine CGSK (Table A-2) has been employed in calculating the data presented in Figures B-2 through B-8. Figure B-2 indicates the maximum deflection angle possible for a given Mach number preceding the shock. The shock angle, and Mach number, temperature, and pressure can be found by use of Figure B-3 through B-6. The entropy increase for small deflection angles is plotted on Figure B-7, and Figure B-8 has the same parameter for larger deflection angles.

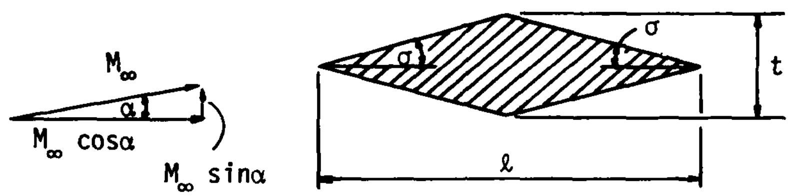
3. OBLIQUE SHOCK WAVE - SWEPT WING



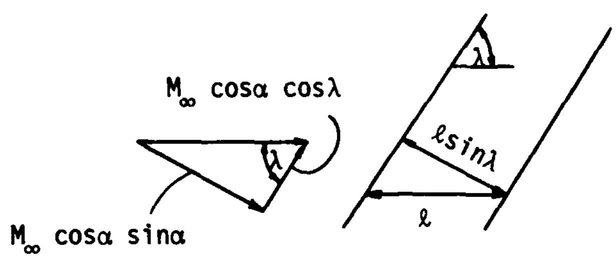
Swept Wing Viewed from Above

While the equations and computer subroutine discussed in Section II.2 are strictly applicable only to two-dimensional situations, it is possible that by appropriate manipulation one can apply them to some problems of flow over a swept wing. Referring to the diagram on the preceding page, two cases are possible with swept wings. In the first case the Mach angle, μ_∞, is greater than the swept angle, λ. In this case the Mach number normal to the leading edge of the wing will be subsonic, and the oblique shock equations cannot be applied. In the second case, μ_∞ is less than λ, M_{∞_n} is supersonic, and the oblique shock equations can be used if equivalent values to use for the upstream Mach number and deflection angle can be determined. The following sketches will be useful for determining these equivalent values.

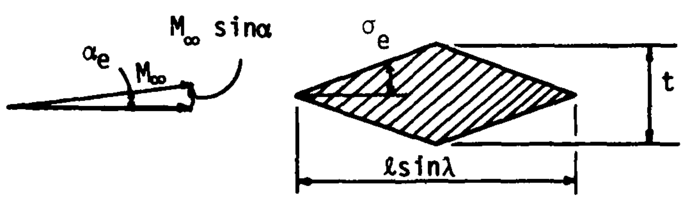
<u>Wing Viewed Parallel to Flight Direction</u>



<u>Wing Viewed from Above</u>



<u>Wing Viewed in Plane Normal to Leading Edge</u>



The equivalent upstream Mach number (M_{∞_e}), can be found from Equation 27 which is derived as follows:

$$M_{\infty_e} = \sqrt{M_\infty^2 \cos^2 \alpha \sin^2 \lambda + M_\infty^2 \sin^2 \alpha} = M_\infty \sqrt{\cos^2 \alpha \sin^2 \lambda + \sin^2 \alpha}$$

$$M_{\infty_e} = M_\infty \sqrt{\cos^2 \alpha \sin^2 \lambda + (1-\cos^2\alpha)} = M_\infty \sqrt{\cos^2 \alpha (\sin^2\lambda - 1) + 1}$$

$$M_{\infty_e} = M_\infty \sqrt{1-\cos^2 \alpha \cos^2\lambda} \tag{27}$$

Similarly, the equivalent angle of attack, α_e, and wing wedge angle, σ_e, can be found as follows:

$$\alpha_e = \tan^{-1}\left(\frac{M_\infty \sin\alpha}{M_\infty \cos \alpha \sin\lambda}\right)$$

$$\alpha_e = \tan^{-1}\left(\frac{\tan \alpha}{\sin \lambda}\right) \tag{28}$$

And:

$$\tan \sigma_e = \frac{t}{\ell \sin \lambda}$$

$$\tan \sigma = t/\ell$$

$$\sigma_e = \tan^{-1}\left(\frac{\tan \sigma}{\sin \lambda}\right) \tag{29}$$

The equivalent deflection angle is

$$\delta_e = \sigma_e + \alpha_e \tag{30}$$

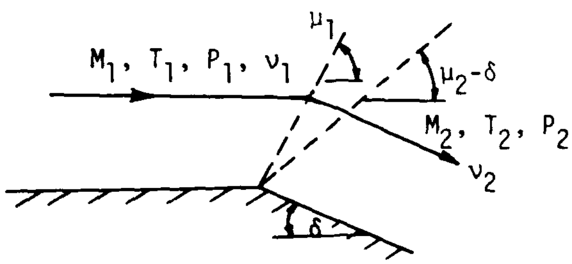
Charts for determining M_{∞_e}/M_∞, α_e and σ_e are included as Figures B-9 and and B-10. Also, a computer subroutine was written to determine the equivalent Mach number and deflection angle for input to subroutine CGSK. The calling statement for the subroutine is

```
CALL SWEP (LAM, M, ALPHA, SIG, MEQ, ALPEQ, SIGEQ)
```

The first four quantities are inputs to the subroutine, and the last three are the desired outputs. Note that the ratio of specific heats does not enter into the calculations of this section. Therefore, the procedures developed in this section and the computer subroutine are applicable to either low or high Mach number supersonic airflows.

See Table A-3 for the listing of subroutine SWEP.

4. ISENTROPIC EXPANSION



A two dimensional airflow encountering a negative deflection or corner will, in an inviscid system, experience an isentropic expansion. The expansion is governed by the well-known Prandtl-Meyer Equation

$$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M^2-1\right)} - \tan^{-1}\sqrt{M^2-1} \tag{31}$$

The Prandtl-Meyer angle, ν, is a property of the airflow and is a function only of the Mach number (for a const γ system). The relationship between Prandtl-Meyer angle upstream and downstream of a corner and the deflection angle is

$$\nu_2 = \nu_1 - (\,-\delta) \tag{32}$$

Therefore, if the Mach number upstream of a corner is known, the corresponding value of ν can be found. Adding to this quantity the deflection angle results in the value of ν downstream of the deflection. From this the corresponding value of Mach number downstream of the deflection can be obtained through an iterative solution of Equation 31.

Since the expansion is isentropic and adiabatic, the static temperature and pressure can be found from the isentropic flow equations

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \tag{33}$$

$$\frac{P_2}{P_1} = \left(\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right)^{\frac{\gamma}{\gamma-1}} \tag{34}$$

Since the flow is isentropic,

$$\frac{\Delta s}{R} = 0$$

And

$$\frac{P_{t_2}}{P_{t_1}} = 1.0$$

An additional parameter generally used for calculations involving isentropic turning is the Mach angle, μ. The Mach angle is simply a function of the Mach number and is defined by the equation

$$\mu = \sin^{-1} \frac{1}{M} \tag{35}$$

The computer subroutine for isentropic turns includes the calculation of Mach angle upstream and downstream of the turn. The calling statement for this subroutine is

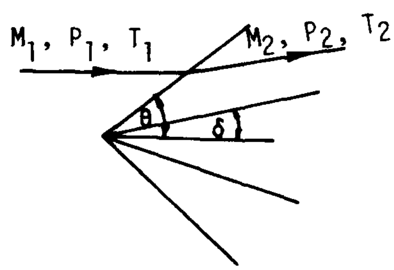
```
CALL EXN (DELT, VM1, T1, P1, GAM, VMU1, VMU2, VM2, T2, P2)
```

The input quantities for this subroutine are: DELT, the deflection angle in degrees; VM1, the upstream Mach number; T1, the upstream static pressure in °R; P1, the upstream static pressure in atmospheres; and GAM,

the ratio of specific heats. For the subroutine to function correctly, the value of DELT must be negative. The parameters determined by use of subroutine EXN are: VMU1, the Mach angle upstream of the deflection in degrees; VMU2, the Mach angle downstream of the deflection, in degrees; VM2, the Mach number downstream; and T2 and P2, the static temperature and pressure downstream. The listing of subroutine EXN is included in Table A-4. A subroutine using Newton's Method is employed to facilitate the iteration procedure. A listing of this program (subroutine NEWT) is included as Table A-5.

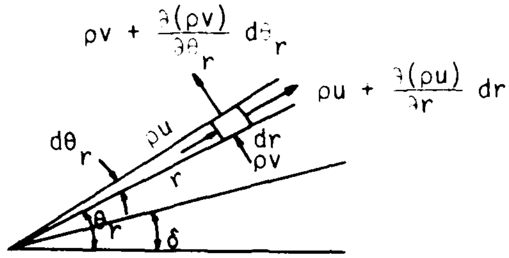
Figures B-11 and B-12 are included to aid in hand calculations of isentropic turning situations. On Figure B-11 the Prandtl-Meyer angle is plotted as a function of Mach number. On Figure B-12 the parameters T/T_t and P/P_t are plotted as functions of Mach number. Since T_t and P_t are constant for this situation, the static temperature and pressure downstream of the deflection can be found once the downstream Mach number has been determined.

5. OBLIQUE SHOCK WAVE - AXISYMMETRIC



The flow over a cone at zero degrees angle of attack is a special case of two-dimensional flow. The two dimensions used to describe the position of a point in space are, in this case, the conical coordinate angle of the point and the distance to the point from the cone tip. The solution for this special case was developed by Taylor-Maccoll. The Taylor-Maccoll Theory is described in numerous references, but will be

repeated here for completeness. The following sketch illustrates the parameters involved with an element of fluid downstream of the shock wave:



For a steadily flowing system, the Continuity Equation for the element of fluid is

$$\rho u \,[r d\theta_r \,(2\pi r \sin\theta_r)] + \rho v \,[dr \,(2\pi r \sin\theta_r)]$$

$$= [\rho u + \frac{\partial(\rho u)}{\partial r} dr][(r+dr)\; d\theta_r (2\pi)(r+dr) \sin\theta_r]$$

$$+ [\rho v + \frac{\partial(\rho v)}{\partial \theta_r} d\theta_r][dr(2\pi r) \sin(\theta_r + d\theta_r)] \tag{36}$$

Multiplying this out and dividing by $2\pi dr d\theta$, we have, after rearrangement,

$$\rho v r \left[\frac{\sin\theta_r}{d\theta_r} - \frac{\sin(\theta_r + d\theta_r)}{d\theta_r}\right] = 2\rho u r \sin\theta_r + \rho u dr \sin\theta_r + \frac{\partial(\rho u)}{\partial r} r^2 \sin\theta_r$$

$$+ \frac{2\partial(\rho u)}{\partial r} r\, dr \sin\theta_r + \frac{\partial(\rho u)}{\partial r} (dr)^2 \sin\theta_r + \frac{\partial(\rho v)}{\partial \theta} r \sin(\theta_r + d\theta_r) \tag{37}$$

As dr approaches 0:

$$\rho u\, dr \sin\theta_r \rightarrow 0$$

$$\frac{2\partial(\rho u)}{\partial r} r\, dr \sin\theta_r \rightarrow 0$$

$$\frac{\partial(\rho u)}{\partial r} (dr)^2 \sin\theta_r \rightarrow 0$$

And as $d\theta_r$ approaches 0:

$$\frac{\sin\theta_r - \sin(\theta_r + d\theta_r)}{d\theta_r} = \frac{\sin\theta_r - \sin\theta_r \cancelto{1}{\cos d\theta_r} - \cos\theta_r \cancelto{d\theta_r}{\sin d\theta_r}}{d\theta_r} \rightarrow$$

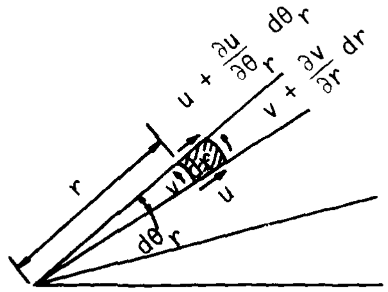
$$\frac{\sin\theta_r - \sin\theta_r - \cos\theta_r d\theta_r}{d\theta_r} = -\cos\theta_r$$

$$\sin(\theta_r + d\theta_r) \rightarrow \sin\theta_r$$

Equation 37 then becomes

$$2\rho ur \sin\theta_r + \frac{\partial(\rho u)}{\partial r} r^2 \sin\theta_r + \frac{\partial(\rho v)}{\partial\theta_r} r \sin\theta_r + \rho vr\cos\theta_r = 0 \qquad (38)$$

The flow in the region between the shockwave and the surface is assumed to be irrotational. This is illustrated by



To keep the crosshatched element from rotating, the following equality must be satisfied:

$$v(rd\theta_r) + \left(u + \frac{\partial u}{\partial\theta_r} d\theta_r\right) dr = u\,dr + (r+dr)\,d\theta_r \left(v + \frac{\partial v}{\partial r} dr\right)$$

Multiplying this out and dividing by $drd\theta_r$, we have

$$\frac{\partial u}{\partial\theta_r} = r\frac{\partial v}{\partial r} + v + \frac{\partial v}{\partial r} dr \qquad (39)$$

As dr approaches zero:

$$\frac{\partial v}{\partial r} dr \rightarrow 0$$

Equation 39 then becomes

$$\frac{\partial u}{\partial \theta_r} - v - r\frac{\partial v}{\partial r} = 0 \qquad (40)$$

The condition that all properties are constant along rays specifies that

$$\frac{\partial}{\partial r} = 0$$

Using this, Equation 38 becomes

$$2\rho ur \sin\theta_r + \frac{d(\rho v)}{d\theta_r} r \sin\theta_r + \rho vr \cos\theta_r = 0 \qquad (41)$$

And, Equation 39 becomes

$$v = \frac{du}{d\theta_r} \qquad (42)$$

After expansion, Equation 41 becomes

$$2\rho ur \sin\theta_r + \rho\frac{dv}{d\theta_r} r \sin\theta_r + v\frac{d\rho}{d\theta_r} r \sin\theta_r + \rho vr \cos\theta_r = 0$$

Dividing by $\rho r \sin\theta_r$, we have

$$2u + \frac{dv}{d\theta_r} + \frac{v}{\rho}\frac{d\rho}{d\theta_r} + v\cot\theta_r = 0$$

Or

$$2u + \frac{dv}{d\theta_r} + v\left(\frac{1}{\rho}\frac{d\rho}{d\theta_r} + \cot\theta_r\right) = 0 \qquad (43)$$

Using Equation 42, Equation 43 can be written

$$\frac{d^2u}{d\theta_r^2} + \frac{du}{d\theta_r}\left(\cot\theta_r + \frac{1}{\rho}\frac{d\rho}{d\theta_r}\right) + 2u = 0 \qquad (44)$$

To make integration of Equation 44 possible, it is necessary to replace the expression involving ρ by one involving u and θ_r only. If we specify that the flow between the shock wave and the surface is isoenergetic, we can use the energy equation to accomplish this. The form of the energy equation to be used is

$$\frac{\gamma}{\gamma-1}\left(\frac{P}{\rho}\right) + \frac{1}{2}\left(u^2 + v^2\right) = \frac{1}{2}\,c^2 \tag{45}$$

Where c is the maximum velocity attainable by expanding the flow to absolute zero temperature. This can be expressed by

$$c = \sqrt{2g\,J\,h_t} = \sqrt{2g\,J\left(h_{SURF} + \frac{u^2_{SURF}}{2gJ}\right)}$$

$$c^2 = 2gJ\,h_{SURF} + u^2_{SURF} \tag{46}$$

Since we have specified that the flow is inviscid, the flow between the shock wave and the surface will also be isentropic. Therefore

$$\frac{P}{\rho^\gamma} = \frac{P_t}{\rho_t^\gamma}$$

And

$$\frac{P}{\rho} = \frac{P}{\rho^\gamma}\left(\rho^{\gamma-1}\right) = \frac{P_t}{\rho_t^\gamma}\left(\rho^{\gamma-1}\right)$$

Using this in Equation 45, we have

$$\frac{\gamma}{\gamma-1}\left(\frac{P_t}{\rho^\gamma_t}\right)\left(\rho^{\gamma-1}\right) + \frac{1}{2}\left(u^2 + v^2\right) = \frac{1}{2}\,c^2 \tag{47}$$

Now, differentiating this with respect to θ_r, we have

$$\frac{\gamma}{\gamma-1}\left(\frac{P_t}{\rho^\gamma_t}\right)(\gamma-1)\left(\rho^{\gamma-2}\right)\frac{d\rho}{d\theta_r} + \frac{1}{2}\left(2u\,\frac{du}{d\theta_r} + 2v\,\frac{dv}{d\theta_r}\right) = 0$$

Or, after rearrangement,

$$\frac{\gamma}{\gamma-1}\left(\frac{P_t}{\rho_t^{\gamma}}\right)\rho^{\gamma-1} = \frac{-u\dfrac{du}{d\theta_r} - v\dfrac{dv}{d\theta_r}}{\dfrac{\gamma-1}{\rho}\dfrac{d\rho}{d\theta_r}} \tag{48}$$

Substituting Equation 48 in Equation 47, we have

$$\frac{-u\dfrac{du}{d\theta_r} - v\dfrac{dv}{d\theta_r}}{\dfrac{\gamma-1}{\rho}\dfrac{d\rho}{d\theta_r}} = \frac{1}{2}\left(c^2 - u^2 - v^2\right)$$

Which can be rearranged to

$$\frac{1}{\rho}\frac{d\rho}{d\theta_r} = \frac{-\left(u\dfrac{du}{d\theta_r} + v\dfrac{dv}{d\theta_r}\right)}{\dfrac{\gamma-1}{2}\left(c^2 - u^2 - v^2\right)}$$

Now, this can be used in Equation 44 to eliminate the dependence on density variation. The resulting equation is

$$\frac{d^2u}{d\theta_r^2} + \frac{du}{d\theta_r}\cot\theta_r - \frac{u\left(\dfrac{du}{d\theta_r}\right)^2 + v\left(\dfrac{dv}{d\theta_r}\right)\left(\dfrac{du}{d\theta_r}\right)}{\dfrac{\gamma-1}{2}\left(c^2 - u^2 - v^2\right)} + 2u = 0 \tag{49}$$

The derivative with respect to θ_r of Equation 42 provides

$$\frac{dv}{d\theta_r} = \frac{d^2u}{d\theta_r^2}$$

Using this, and Equation 42 in Equation 49, we have

$$\frac{d^2u}{d\theta_r^2} + v\cot\theta_r - \frac{uv^2 - v^2\left(\dfrac{d^2u}{d\theta_r^2}\right)}{\dfrac{\gamma-1}{2}\left(c^2 - u^2 - v^2\right)} + 2u = 0$$

Which becomes, after collecting terms

$$\frac{d^2u}{d\theta_r^2}\left[\frac{\gamma+1}{2}v^2-\frac{\gamma-1}{2}(c^2-u^2)\right]=(\gamma-1)(c^2-u^2)u+\frac{\gamma-1}{2}(c^2-u^2)\cot\theta_r\, v-\gamma uv^2-\frac{\gamma-1}{2}\cot\theta_r v^3$$

This can be nondimensionalized with c to yield

$$\frac{d^2\left(\frac{u}{c}\right)}{d\theta_r^2}=\frac{(\gamma-1)\left(1-\frac{u^2}{c^2}\right)\frac{u}{c}+\frac{\gamma-1}{2}\left(1-\frac{u^2}{c^2}\right)\cot\theta_r\left(\frac{v}{c}\right)-\gamma\left(\frac{u}{c}\right)\left(\frac{v}{c}\right)^2-\frac{\gamma-1}{2}\cot\theta_r\left(\frac{v}{c}\right)^3}{\frac{\gamma+1}{2}\left(\frac{v}{c}\right)^2-\frac{\gamma-1}{2}\left(1-\frac{u^2}{c^2}\right)} \tag{50}$$

Equation 50 can be solved by numerical integration in an inverse manner. The integration proceeds from the cone surface to the shock wave, and provides properties of the airflow on rays emanating from the cone apex. The technique requires that the cone half angle and the velocity at the cone surface are known. The calculated parameters are then the upstream Mach number, and the shock wave angle.

At the cone surface

$$\theta_r = \delta$$

$$\frac{u}{c} = \frac{u_{SURF}}{c}$$

$$\frac{d\left(\frac{u}{c}\right)}{d\theta_r} = \frac{v}{c} = 0$$

Using numerical integration the first derivative on any ray can be expressed approximately as

$$\left(\frac{d\frac{u}{c}}{d\theta_r}\right)_{\theta_r+\Delta\theta_r}=\left(\frac{d\frac{u}{c}}{d\theta_r}\right)_{\theta_r}+\Delta\theta_r\left(\frac{d^2\frac{u}{c}}{d\theta_r^2}\right)_{\theta_r} \tag{51}$$

Now, if this value of the first derivative is averaged with the preceding value, and added to the preceding value of u/c, we can find the new value of u/c.

$$\left(\frac{u}{c}\right)_{\theta_r+\Delta\theta_r} = \left(\frac{u}{c}\right)_{\theta_r} + \frac{\left(\dfrac{d\,\frac{u}{c}}{d\theta_r}\right)_{\theta_r}\Delta\theta_r + \left[\left(\dfrac{d\,\frac{u}{c}}{d\theta_r}\right)_{\theta_r} + \Delta\theta_r\left(\dfrac{d^2\,\frac{u}{c}}{d\theta_r^2}\right)_{\theta_r}\right]\Delta\theta_r}{2}$$

$$\left(\frac{u}{c}\right)_{\theta_r+\Delta\theta_r} = \left(\frac{u}{c}\right)_{\theta_r} + \Delta\theta_r\left(\frac{d\,\frac{u}{c}}{d\theta_r}\right)_{\theta_r} + \frac{(\Delta\theta_r)^2}{2}\left(\frac{d^2\,\frac{u}{c}}{d\theta_r^2}\right)_{\theta_r} \tag{52}$$

With Equation 52, all that is required to determine the parameters on a particular ray is data on the preceding ray. Therefore, it is possible to proceed from the ray on the cone surface (where the data is known) to the ray on the shock wave.

An example of the numerical integration procedure will illustrate its features. For the example, the following data will be set:

$$\delta = 10°,\ \frac{u_{SURF}}{c} = .8,\ \gamma = 1.4,\ \Delta\theta_r = 1° = .01745 \text{ rad.}$$

At the surface the first derivative is zero (from Equation 42). The second derivative is found from Equation 50:

$$\left(\frac{d^2\,\frac{u}{c}}{d\theta_r^2}\right)_\delta = \frac{.4(.36)(.8)+.2(.36)\cot 10°(0)-1.4(.8)(0)^2-.2(\cot 10°)(0)^3}{1.2(0)^3-.2(.36)} = -1.6$$

Then proceding to the next ray (at θ=11°) and, using Equation 51,

$$\left(\frac{d\,\frac{u}{c}}{d\theta_r}\right)_{\theta_r} = 0 + .01745(-1.6) = -.027925$$

And, using Equation 52,

$$\left(\frac{u}{c}\right)_{\theta_{r_1}} = .8 + .01745(0) + \frac{(.01745)^2}{2}(-1.6) = .799756$$

Again, using Equation 50,

$$\left(\frac{d^2\frac{u}{c}}{d\theta_r^2}\right)_{\theta_{r_1}} = \frac{.4(.36039)(.799756)+.2(.36039)(\cot 11°)(-.027925)-1.4(.799756)(-.027925)^2}{1.2(-.027925)^2 - .2(.36039)} - \frac{.2(\cot 11°)(-.027925)^3}{1.2(-.027925)^2 - .2(.36039)}$$

$$\left(\frac{d^2\frac{u}{c}}{d\theta_r^2}\right)_{\theta_{r_1}} = -1.46304$$

This procedure is repeated for successive rays until the assumed shock wave location is passed. For each ray it is convenient to determine the resultant flow angle, velocity and Mach number. These can be found by use of Equations 53 through 55:

$$\phi = \tan^{-1}\frac{v/c}{u/c} + \theta_r \tag{53}$$

$$w/c = \sqrt{(v/c)^2 + (u/c)^2} \tag{54}$$

$$M = \sqrt{\frac{2}{\gamma-1}\left[\frac{(w/c)^2}{1-(w/c)^2}\right]} \tag{55}$$

A summary of the calculated data for rays out to 22° is presented in Table 1 for the example case.

TABLE 1

SUMMARY OF CALCULATIONS FOR CONE FLOW NUMERICAL INTEGRATION EXAMPLE

θ_r	u/c	v/c	w/c	ϕ	M
10°	.8	0	.8	10°	2.981424
11°	.799756	-.027925	.800243	9.000°	2.983942
12°	.799046	-.053460	.800832	8.172°	2.990061
13°	.797904	-.0773589	.801645	7.462°	2.998547
14°	.796356	-.100101	.802623	6.836°	3.008818
15°	.794418	-.122021	.803734	6.268°	3.020573
16°	.792102	-.143379	.804974	5.740°	3.033787
17°	.789416	-.164404	.806354	5.236°	3.048629
18°	.786364	-.185340	.807910	4.738°	3.065546
19°	.782944	-.206522	.809724	4.220°	3.085486
20°	.779147	-.228584	.811986	3.650°	3.110717
21°	.774941	-.253496	.815349	2.886°	3.149000
22°	.770382	-.268989	.815992	2.753°	3.156426

Despite the fact that rays out to $\theta_r = 22°$ have been calculated, we as yet have no indication of the location of the shock wave. Data calculated for rays upstream of the shock wave is not valid. To find the shock wave angle, we use the fact that the flow deflection through a conical shock wave is the same as that for an oblique shock wave with the same upstream conditions. Also, the ratio of upstream Mach number to Mach number just downstream of the shock wave will be the same for both conical and oblique shock waves. Therefore, it is possible to use Figure B-4, which was plotted for oblique shock waves, to determine the Mach number preceding the conical shock wave. Then Figure B-3 can be used to determine the shock angle for the conical system.

To demonstrate this, the example case as summarized in Table 1 will be continued. To obtain a result of higher precision than the calculation increment of $\Delta\theta_r$, a trial and error solution will be employed. As a first guess, we shall assume the shock wave angle to be 19°. For this ray (from Table 1) the flow deflection angle is 4.220°. Using this, and Figure B-3, we find, with $\delta/\theta = 0.1936$, that $M_1 = 3.53$. Now, using this, the deflection angle, and Figure B-4, we find that $M_2 = 0.935\ (3.53) = 3.300$. Checking this value with the value of M tabulated in Table 1, we see that it is too high. Therefore, the next guess for the shock wave angle is 20°. The results using this guess are: $M_1 = 3.335$, $M_2 = 3.135$. Again, this value of M_2 is too high, so the next guess for the shock wave angle is 21°. Using this guess, the results are: $M_1 = 3.077$, $M_2 = 2.923$. This value of M_2 is too low, so we know that the shock wave angle lies somewhere between 20° and 21°. To obtain a more precise result, we could repeat the numerical integration between $\theta_r = 20°$ and $\theta_r = 21°$ with a smaller step size. An alternate method is to simply interpolate to obtain the shock wave angle. Using this method we find:

$$\theta = 20 + \frac{1(3.135-3.1107)}{(3.135-3.1107) + (3.149-2.923)} = 20.1°$$

$$M_2 = 3.1107 + (3.1490-3.1107)\left[\frac{(3.135-3.1107)}{(3.135-3.1107) + (3.149-2.923)}\right] = 3.1144$$

$$M_2 = 3.335-(3.335-3.077)\left[\frac{(3.135-3.1107)}{(3.135-3.1107) + (3.149-2.923)}\right] = 3.310$$

A computer program written to perform the calculations illustrated in the preceding example is included as Table A-6. The input data to this program are: DELT, the cone half angle in degrees; USC, the surface velocity, normalized; GAM, the ratio of specific heats; and, DTHETA, the calculation interval in degrees. The output for one set of input data is on a single sheet. A sample output is included as Figure 1. The input for this sample is the same as for the preceding example: DELT = 10., USC = 0.8, GAM = 1.4, and DTHETA =1.

CONE FLOW CALCULATION

(GAMMA = 1.4[illegible])

CONE HALF ANGLE = 1[illegible].[illegible] DEGREES

USURF/C = .8[illegible]

THETA-DEG	V/C	U/C	W/C	PHI-DEG	M2
11.[illegible]	-.[illegible]279	.799[illegible]	.8[illegible]2	9.[illegible]	2.98[illegible]
12.[illegible]	-.[illegible]35	.739[illegible]	.8[illegible]3	[illegible].17	2.99[illegible]
13.[illegible]	-.[illegible]74	.7979	.8[illegible]15	7.46	2.999
14.[illegible]	-.1[illegible]1	.7964	.8[illegible]2[illegible]	[illegible].64	3.0[illegible]9
15.[illegible]	-.122[illegible]	.7944	.8[illegible]37	5.27	3.[illegible]21
16.[illegible]	-.1[illegible]34	.7921	.855[illegible]	5.74	3.[illegible]34
17.[illegible]	-.1[illegible]44	.7[illegible]94	.8[illegible]	5.24	3.[illegible]4[illegible]
18.[illegible]	-.1[illegible]55	.7664	.8[illegible]73	4.74	3.[illegible]06
19.[illegible]	-.[illegible]	.7823	.8[illegible]97	4.22	3.[illegible]
2[illegible]	-.[illegible]8[illegible]	.7791	.812[illegible]	3.65	3.111

M1 = [illegible].31[illegible]9 THETA S = [illegible].075DEG

(P2/P1)SHK = 1.344 (P2/P1)SURF = 1.[illegible]37

(T2/T1)SHK = 1.[illegible] (T2/T1)SURF = 1.1[illegible]2

DS/R = .[illegible]026

Figure 1. Sample Output for Program CONE

The effect of the calculation interval has been investigated for a case with a cone half angle of 20°, a normalized surface velocity of 0.6, and γ=1.4. The results of this investigation are shown in Figure 2, where each of the calculated parameters is plotted versus $\Delta\theta_r$. Obviously, the smaller the calculation interval, the more nearly correct the numerical integration results will be. However, there is a practical limit, even with a high-speed computer, to the size of the interval. On Figure 2 a point has been placed on each of the lines indicating the value of $\Delta\theta_r$ that will result in an error of 0.1%. Thus for calculation of shock angle, a $\Delta\theta_r$ = 1° will probably be sufficient in most cases. However, if the value of one of the other parameters is needed with great accuracy, a value of $\Delta\theta_r$=0.5° or less may be needed.

In most cases, the data available is the upstream flow conditions and the cone angle. The calculation technique described is an inverse technique, and therefore must be iterated in order to accommodate the usually available data. A computer subroutine has been written to accomplish this. The calling statement for the conical shock computer subroutine is:

```
CALL CONSK (VM1, DELT, GAM, THE, THETAS, DSR, VM2C, PHIC, PRC, TRC,
VM2S, PRSHK, TRSHK, PHIS, VM2SRF, PRSRF, TRSRF)
```

The first four items in this sequence are inputs. VM1 is the upstream Mach number, DELT is the cone half angle in degrees, and GAM is the ratio of specific heats. The fourth input has been included to provide data on one specific ray lying between the shock wave and the surface. THE is the ray angle, in degrees, for which data is required. The output information is as follows: THETAS is the shock wave angle, DSR is the non-dimensionalized entropy across the shock wave, VM2C is the Mach number on the specified ray, PHIC is the flow angle on the specified ray, PRC is the ratio of the static pressure on the specified ray to the upstream static pressure, TRC is the ratio of the static temperature on the specified ray to the upstream static pressure, VM2S is the Mach number just downstream of the shock wave, PRSHK is the ratio of static pressure across the shock wave, TRSHK is the ratio of static temperature across the shock wave, PHIS is the flow angle just downstream of the shock wave, VM2SRF is the Mach number on the cone surface, PRSRF is the ratio of the

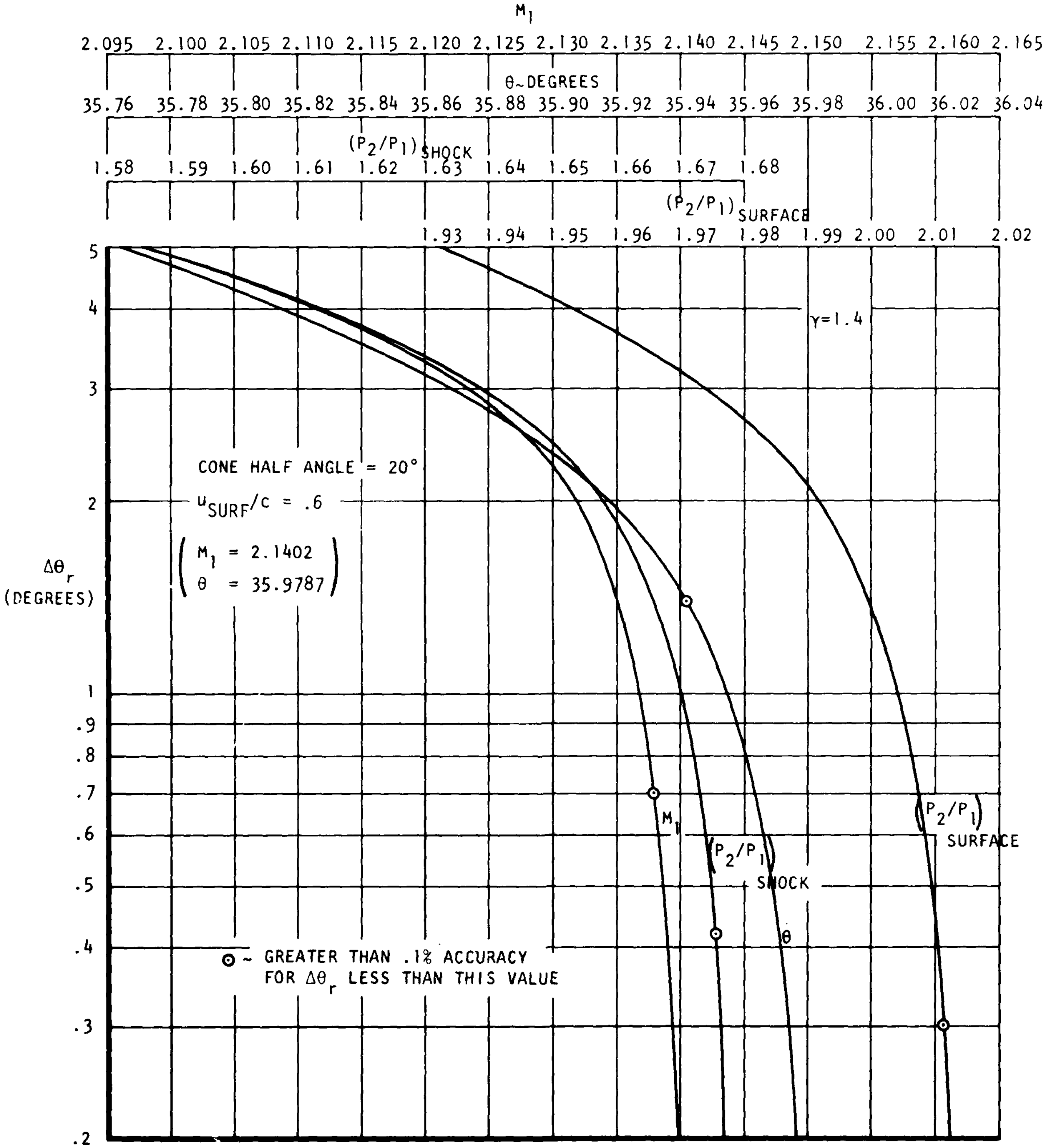


Figure 2. Effect of Calculation Increment on Accuracy of Cone Flow Calculation Procedure

static pressure on the surface to the upstream static pressure, and TRSRF is the ratio of the static temperature on the surface to the upstream static temperature. A listing of subroutine CONSK is included as Table A-7. Subroutine NEWT is also required in conjunction with subroutine CONSK. Subroutine CONSK has been employed in calculating the data presented in Figures B-13 through B-17. Figure B-13 can be used to find the shock wave angle if the upstream Mach number and cone half angle are known. Figure B-14 presents the Mach number ratio across the shock wave and ratio of surface Mach number to upstream Mach number. Figures B-15 and B-16 can be used to find the ratio of upstream static temperature or pressure to the corresponding parameter just downstream of the shock wave, or on the cone surface. Figure B-17 presents the non-dimensionalized entropy increase across the shock wave. The data plotted in these charts was calculated with a calculation interval of 0.1 degree. The results, then, are quite precise.

To illustrate the use of subroutine CONSK an additional program has been written. The listing of this program, which is entitled program CONES, is included as Table A-8. The input data to this program are: VM1, the upstream Mach number; DELT, the cone half angle in degrees; GAM, the ratio of specific heats; and THE, the angle in degrees of a ray for which data is desired. A sample output for program CONES is included as Figure 3. The input data used to generate Figure 3 is as follows: VM1=3.0, DELT=20., GAM=1.4, THE=25.

```
FLOW CALCULATION
GAMMA = 1.[illegible]

                    [illegible] = [illegible]

[illegible] = 2[illegible].[illegible]

[illegible] = [illegible]

[illegible] = 2[illegible]. [illegible]

[illegible] 2.3[illegible]7
PHI = [illegible].[illegible]DEG
P2/P1 = [illegible].[illegible]
T2/T1 = 1.3[illegible]

[illegible]

M[illegible] = [illegible]
P[illegible]/P1 = [illegible]
T[illegible]/T1 = [illegible].3[illegible]

AT SHOCK:

M2 = 2.3[illegible]7
PHI = 12.3[illegible]DEG
P2/P1 = [illegible].3[illegible]59
T2/T1 = 1.3[illegible]35
```

Figure 3. Sample Output for Program CONES

SECTION III

TECHNIQUES FOR HIGH MACH NUMBER (M>4) SUPERSONIC AIRFLOWS

At flight Mach numbers greater than about Mach 4, the temperature of the air being decelerated by a propulsion system inlet is increased to such an extent that the equations utilized in the techniques described in Section II are no longer applicable. The Section II equations are generally based on the assumption that the air is calorically perfect, and the specific heats are constant. In this Section the conservation equations will be employed to provide "real gas" techniques for determining conditions downstream of the various flow pehnomena.

The real gas calculation techniques generally require iterative solutions and are more complicated than those discussed in Section II. Computer solutions are required for all but a few sample cases. It is not possible to prepare charts for use as computation aids, as were presented for the Section II techniques.

Computer subroutines are presented for each of the flow phenomena discussed in this Section. All of these subroutines employ an additional subroutine that provides the equilibrium thermodynamic properties of air. This subroutine is based on the approximate model for equilibrium air presented by Hansen and Hodge in Reference 1. A listing of subroutine THAIR is included as Table A-9. The equations used in this subroutine will not be repeated here since they are taken directly from Reference 1. The use of the subroutine, however, will be discussed. The calling statement for the equilibrium air model subroutine is

```
CALL THAIR (T, P, H, S, RHO, AM, CP, CV, GAM, A, K1, K2)
```

In this sequence:

T is temperature in °R.
P is pressure in atmospheres.
H is enthalpy in Btu per pound.
S is entropy in Btu per pound °R.
RHO is density in pounds per cubic foot.
AM is molecular weight.

CP is the specific heat at constant pressure in Btu per pound °R.

CV is the specific heat at constant volume in Btu per pound °R.

GAM is the local isentropic exponent, defined as the partial derivative of the logarithm of pressure with respect to the logarithm of density, at constant entropy. In equation form, this is

$$\gamma_I = \left(\frac{\partial \log p}{\partial \log \rho}\right)_s$$

(At temperatures less than about 1000°R, the isentropic exponent and the ratio of specific heats have very nearly equal values).

A is the speed of sound in feet per second.

K1 and K2 are operational inputs.

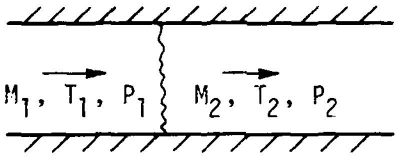
Subroutine THAIR has a number of options, exercised by the values assigned to K1 and K2. The parameter K1 may be assigned a value from 0 to 4. The effect of each value of K1 can be shown in tabular form:

If K1=0 → Input T and P; Calculate H, S, RHO, AM
If K1=1 → Input H and P; Calculate T, S. RHO, AM
If K1=2 → Input S and P; Calculate T, H, RHO, AM
If K1=3 → Input S and T; Calculate P, H, RHO, AM
If K1=4 → Input S and H; Calculate T, P, RHO, AM

The parameter K2 may be assigned a value of zero, one, or two. The use of the default condition, K2=0, results in the calculations dependent on the value of K1, as listed above, and values are not determined for CP, CV, GAM, and A. If K2=1, the calculations dependent on the value of K1 are made, and in addition the values of CP, CV, GAM, and A are determined. with K2=2, the value of GAM must be an input quantity, and, depending on the value of K1, all the other quantities are determined on a constant gamma basis. This option is particularly useful when developing or checking out a new computer program or subroutine. The use of the K2=2 option dramatically reduces the computer time used. The K2=1 option is also provided to reduce computer time. The calculation of CP, CV, GAM, and A takes a significant amount of computer time. If these quantities are not required for a particular calculation, the computer time can be reduced by setting K2=0 and, thereby, eliminating the additional computation.

More precise programs for determining the thermodynamic properties of air are available. However, the use of these programs, particularly for iterative solutions, can be quite costly in computer time. Subroutine THAIR is a good compromise between precision and cost. Comparison of data generated by subroutine THAIR and data in standard references, such as Reference 6 indicates very good agreement at temperatures up to 6000°R. This should be adequate for most calculations of practical interest.

1. NORMAL SHOCK WAVE



For any flow process the mass, momentum, and energy must be conserved. If the flow is steady and adiabatic the conservation equations for the normal shock process shown above can be expressed as follows:

Energy Conservation: $$h_2 + \frac{V_2^2}{2gJ} = h_1 + \frac{V_1^2}{2gJ} \tag{56}$$

Momentum Conservation: $$P_2 + \frac{\rho_2 V_2^2}{g} = P_1 + \frac{\rho_1 V_1^2}{g} \tag{57}$$

Mass Conservation: $$\rho_2 V_2 = \rho_1 V_1 \tag{58}$$

The number of equations can be increased to the number of unknown quantities by applying the definition of Mach number, and the thermodynamic properties of air as follows:

$$M_2 = \frac{V_2}{a_2} \tag{59}$$

$$M_1 = \frac{V_1}{a_1} \tag{60}$$

$$h_1 = f(T_1, P_1) \tag{61}$$

$$a_1 = f(T_1, P_1) \tag{62}$$

$$\rho_1 = f(T_1, P_1) \tag{63}$$

$$h_2 = f(T_2, P_2) \tag{64}$$

$$a_2 = f(T_2, P_2) \tag{65}$$

$$\rho_2 = f(h_2, P_2) \tag{66}$$

Generally, M_1, T_1, and P_1 upstream of the shock wave will be known, and the corresponding properties downstream of the shock wave will need to be calculated. The calculation procedure to be used in this case is as follows:

Step 1 - Find h_1, a_1, ρ_1 using Equations 61, 62, and 63

Step 2 - Find V_1 using Equation 60

Step 3 - Assume a value for ρ_2

Step 4 - Find V_2 using Equation 58

Step 5 - Find P_2 using Equation 57

Step 6 - Find h_2 using Equation 56

Step 7 - Check the assumed value of ρ_2 by use of Equation 66

Step 8 - If the calculated value of ρ_2 (Step 7) is not close enough to the assumed value of ρ_2 (Step 3), assume a new value of ρ_2 and repeat Steps 4-7 until the assumed and calculated values are close enough. Then proceed to Step 9.

Step 9 - Find T_2 and a_2 using Equations 64 and 65

Step 10 - Find M_2 using Equation 59

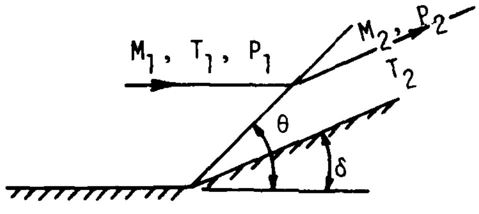
Steps 1, 7, and 9 will require a Mollier Diagram for equilibrium air if hand calculations are performed with this calculation procedure. Such a diagram is contained in Reference 6.

A subroutine for calculating on a real gas basis the properties downstream of a normal shock has been written. The calling statement for this subroutine is:

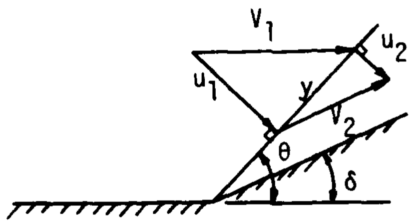
```
CALL NRMSK (VM1, T1, P1A, VM2, T2, P2A, GAM)
```

The input items for this subroutine are VM1, T1, P1A, and GAM. VM1 is the upstream Mach number, T1 is the upstream static pressure in °R, and P1A is the upstream static pressure in atmospheres. The value of GAM is zero if the real gas solution is desired. If a non-zero value of GAM is input, the calculations will be done on a perfect gas basis using the input value of GAM as the ratio of specific heats. This option is included for conservation of computer time while developing new programs and subroutines. VM2, T2, and P2A are the calculated parameters, and have the same units as the corresponding input parameters. A listing of subroutine NRMSK is included as Table A-10. The use of subroutine THAIR is required with subroutine NRMSK.

2. OBLIQUE SHOCK WAVE-PLANAR



The problem of supersonic flow through a two-dimensional deflection can be addressed by applying the conservation equations in a direction perpendicular to the shock wave. A vector diagram is helpful for illustrating this:



Then, assuming an adiabatic system, the conservation equations perpendicular to the shock wave are

$$h_2 + \frac{u_2^2}{2gJ} = h_1 + \frac{u_1^2}{2gJ} \tag{67}$$

$$P_2 + \frac{\rho_2 \, u_2^2}{g} = P_1 + \frac{\rho_1 \, u_1^2}{g} \tag{68}$$

$$\rho_2 u_2 = \rho_1 u_1 \tag{69}$$

From the vector diagram:

$$u_1 = V_1 \sin \theta \tag{70}$$

The upstream Mach number is related to the velocity by

$$M_1 = \frac{V_1}{a_1} \tag{71}$$

Where the speed of sound is

$$a_1 = \sqrt{g\gamma_1 \, R_1 \, T_1} \tag{72}$$

Combining Equations 70 through 72:

$$u_1^2 = M_1^2 \sin^2 \theta (g\gamma_1 \, R_1 \, T_1) \tag{73}$$

Equation 67 can be arranged to

$$h_1 \left(\frac{h_2}{h_1} - 1 \right) = \frac{u_1^2}{2gJ} \left(1 - \frac{u_2^2}{u_1^2} \right)$$

Then, substituting with Equation 73:

$$h_1 \left(\frac{h_2}{h_1} - 1 \right) = \frac{M_1^2 \sin^2 \theta (g\gamma_1 \, R_1 \, T_1)}{2gJ} \left(1 - \frac{u_2^2}{u_1^2} \right) \tag{74}$$

Although this is a real-gas solution, one can validly use relations involving the specific heat and the isentropic exponent as long as the use is restricted to one side or the other of the shock wave. The use of similar relations across the shock wave would not be valid. The upstream temperature and enthalpy are therefore related by

$$T_1 = \frac{h_1}{c_{p_1}}$$

Where

$$c_{p_1} = \frac{\gamma_1 \; R_1}{(\gamma_1 - 1)J}$$

Equation 74 can then be written

$$\frac{h_2}{h_1} - 1 = \frac{M_1^2 \sin^2 \theta(\gamma_1 \; R_1 \; T_1)}{2J \; c_{p_1} \; T_1} \left(1 - \frac{u_2^2}{u_1^2}\right)$$

$$\frac{h_2}{h_1} - 1 = \frac{M_1^2 \sin^2 \theta(\gamma_1 - 1)}{2} \left(1 - \frac{u_2^2}{u_1^2}\right) \tag{75}$$

Now, rearranging Equation 68:

$$P_1 \left(\frac{P_2}{P_1} - 1\right) = \frac{\rho_1 u_1^2 - \rho_2 u_2^2}{g}$$

$$\frac{P_2}{P_1} - 1 = \frac{u_1^2 \left(\rho_1 - \rho_2 \frac{u_2^2}{u_1^2}\right)}{P_1 g}$$

Substituting for $u_1{}^2$ with Equation 73:

$$\frac{P_2}{P_1} - 1 = \frac{M_1^2 \sin^2 \theta(g\gamma_1 \; R_1 \; T_1)\left(\rho_1 - \rho_2 \frac{u_2^2}{u_1^2}\right)}{P_1 g} \tag{76}$$

The Perfect Gas Law is valid for use as long as the application is for conditions on only one side of the shock wave. Therefore,

$$P_1 = \rho_1 R_1 T_1$$

Using this in Equation 76:

$$\frac{P_2}{P_1} - 1 = \frac{M_1^2 \sin^2 \theta(\gamma_1) \left(\rho_1 - \rho_2 \frac{u_2^2}{u_1^2}\right)}{\rho_1}$$

Using Equation 69 (the Conservation of Mass equation):

$$\frac{P_2}{P_1} - 1 = M_1^2 \sin^2 \theta(\gamma_1) \left[1 - \frac{u_1}{u_2} \left(\frac{u_2}{u_1}\right)^2\right]$$

Which becomes

$$\frac{P_2}{P_1} - 1 = \gamma_1 M_1^2 \sin^2 \theta \left(1 - \frac{u_2}{u_1}\right) \tag{77}$$

Again referring to the vector diagram on page 34:

$$\tan (\theta - \delta) = \frac{u_2}{y}$$

$$\tan \theta = \frac{u_1}{y}$$

And

$$\frac{\tan (\theta - \delta)}{\tan \theta} = \frac{u_2}{u_1} \tag{78}$$

Additional equations that are required to match the number of unknowns can be obtained by application of the definition of Mach number, the vector diagram on page 34, and the thermodynamic properties of air. These equations are

$$h_1 = f\,(T_1,\ P_1) \tag{79}$$

$$\rho_1 = f\,(T_1,\ P_1) \tag{80}$$

$$\rho_2 = f\,(T_2,\ P_2) \tag{81}$$

$$u_2 = V_2 \sin\,(\theta - \delta) \tag{82}$$

$$M_2 = V_2\ a_2 \tag{83}$$

$$a_1 = f\,(T_1,\ P_1) \tag{84}$$

$$a_2 = f\,(T_2,\ P_2) \tag{85}$$

$$T_2 = f\,(h_2,\ P_2) \tag{86}$$

$$\gamma_1 \equiv \left(\frac{\partial \log P_1}{\partial \log \rho_1}\right)_s = f\,(T_1,\ P_1) \tag{87}$$

Inclusion of Equations 79 through 87 matches the number of unknowns with the number of applicable equations. A solution is therefore possible. The calculation procedure to be used is as follows:

Step 1: Find h_1, ρ_1, a_1, and γ_1, using Equations 79, 80, 84, and 87.

Step 2: Assume a value for the shock wave angle, θ (this value must be reasonably correct in order to insure that the weak shock solution will be determined). The shock wave angle found though an ideal-gas solution will generally be a good value for the first assumption).

Step 3: Find u_2/u_1, using Equation 78.

Step 4: Find P_2, using Equation 77.

Step 5: Find h_2, using Equation 75.

Step 6: Find ρ_2, using Equation 81.

Step 7: Check the value of u_2/u_1 calculated in Step 3 with a value calculated by use of Equation 69.

Step 8: If the value of u_2/u_1 calculated in Step 7 is not close enough to the value calculated in Step 3, assume a new value for the shock wave angle, and repeat Steps 3-7 until the two values of u_2/u_1 are c'ose enough. Then proceed to Step 9.

Step 9: Find T_2 and a_2, using Equations 85 and 86.

Step 10: Find V_1, using Equation 71.

Step 11: Find u_1, using Equation 70.

Step 12: Fund u_2, using the relation: $u_2 = u_1 \; (u_2/u_1)$.

Step 13: Find V_2, using Equation 82.

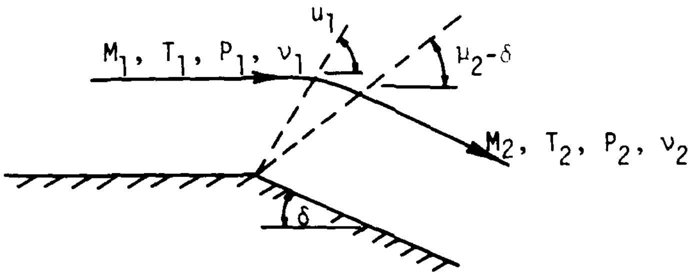
Step 14: Find M_2, using Equation 83.

A subroutine using this calculation procedure to determine the conditions downstream of an oblique shock wave, given the upstream conditions and the deflection angle, has been written. The calling statement for this subroutine is:

```
CALL RGSK (DELTA, VMA, TA, PA, THETA, VMB, TB, PB)
```

The input items for subroutine RGSK are DELTA, VMA, TA, and PA. DELTA is the flow deflection in degrees, VMA is the upstream Mach number, TA is the upstream static temperature in °R, and PA is the upstream static pressure in atmospheres. THETA is the shock wave angle in degrees. VMB, TB, and PB are the calculated parameters downstream of the shock wave, and have the same units as the corresponding input parameters. The information determined in subroutine RGSK is based on the weak shock solution which is generally the solution of interest. No provision is made to provide the strong shock solution in this subroutine. A listing of subroutine RGSK is included as Table A-11. The use of subroutines THAIR, NEWT, and CGSK are required with subroutine RGSK.

3. ISENTROPIC EXPANSION



The real gas solution for a two dimensional isentropic expansion is obtained in a manner similar to that employed for the ideal gas solution. However, since the value of the isentropic exponent (γ_I) varies throughout the expansion, the calculation procedure must be carried out in a number of small steps.

The applicable equations are

$$\nu = \sqrt{\frac{\gamma_I+1}{\gamma_I-1}}\ \tan^{-1}\sqrt{\frac{\gamma_I-1}{\gamma_I+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1} \tag{88}$$

$$\nu_{i+1} = \nu_1 + \delta_i \tag{89}$$

$$\frac{T_2}{T_1} = \frac{1+\frac{\gamma_I-1}{2}M_1^2}{1+\frac{\gamma_I-1}{2}M_2^2} \tag{90}$$

$$\frac{P_2}{P_1} = \left(\frac{1+\frac{\gamma_I-1}{2}M_1^2}{1+\frac{\gamma_I-1}{2}M_2^2}\right)^{\frac{\gamma_I}{\gamma_I-1}} \tag{91}$$

Using these equations, and knowing M_1, T_1, P_1, and s, the calculation procedure is as follows: .

Step 1: Divide δ into a sufficient number of equal increments "n".

Step 2: Find $\gamma_{I_1} = f\ (T_1,\ P_1)$

Step 3: Find ν_1, using Equation 88

Step 4: Find ν_{i+1}, using Equation 89

Step 5: Find M_{i+1}, using an iterative solution with Equation 88

Step 6: Find $T_{2_{i+1}}$ and $P_{2_{i+1}}$, using Equations 90 and 91

Step 7: If i<n find $\gamma_{I_{i+1}} = (T_{2_{i+1}})$ and repeat Steps 2-6 until i=n. The desired values of M_2, T_2, and P_2 have then been determined.

A subroutine using this calculation procedure was written to calculate, on a real gas basis, the conditions downstream of an isentropic expansion. The calling statement for this subroutine is:

```
CALL RGEXP(T1,P1,VM1,DEFL,DEFLN,GAM,VMU1,VMU2,T2,P2,VM2)
```

The parameters input to subroutine RGEXP are T1, P1, VM1, DEFL, DEFLN, and GAM. If the calculations are to be accomplished on a real gas basis, GAM is set equal to zero. If ideal gas calculations are desired, GAM is set equal to the value of the ratio of specific heats. T1 is the upstream temperature in °R, P1 is the upstream pressure in atmospheres, VM1 is the upstream Mach number, and DEFL is the deflection angle in degrees. DEFL is always a negative number. DEFLN is the number of equal increments that the deflection is to be divided into in order to provide results of sufficient accuracy. Figure 4 indicates the effect on accuracy of various values for DEFLN. This data was obtained for a real gas case with VM1=6.0, T1=2500°R, and P1=.01 atmospheres. The total deflection, DEFL, is 15°, and the resulting values of M_2, T_2, and P_2 are shown as calculated using various values of DEFLN. For the Mach 6 case shown, values of DEFLN that produce incremental deflections less than about .5° provide results that are generally adequate. For Mach numbers greater than 6 where the value of the isentropic exponent changes more rapidly, a

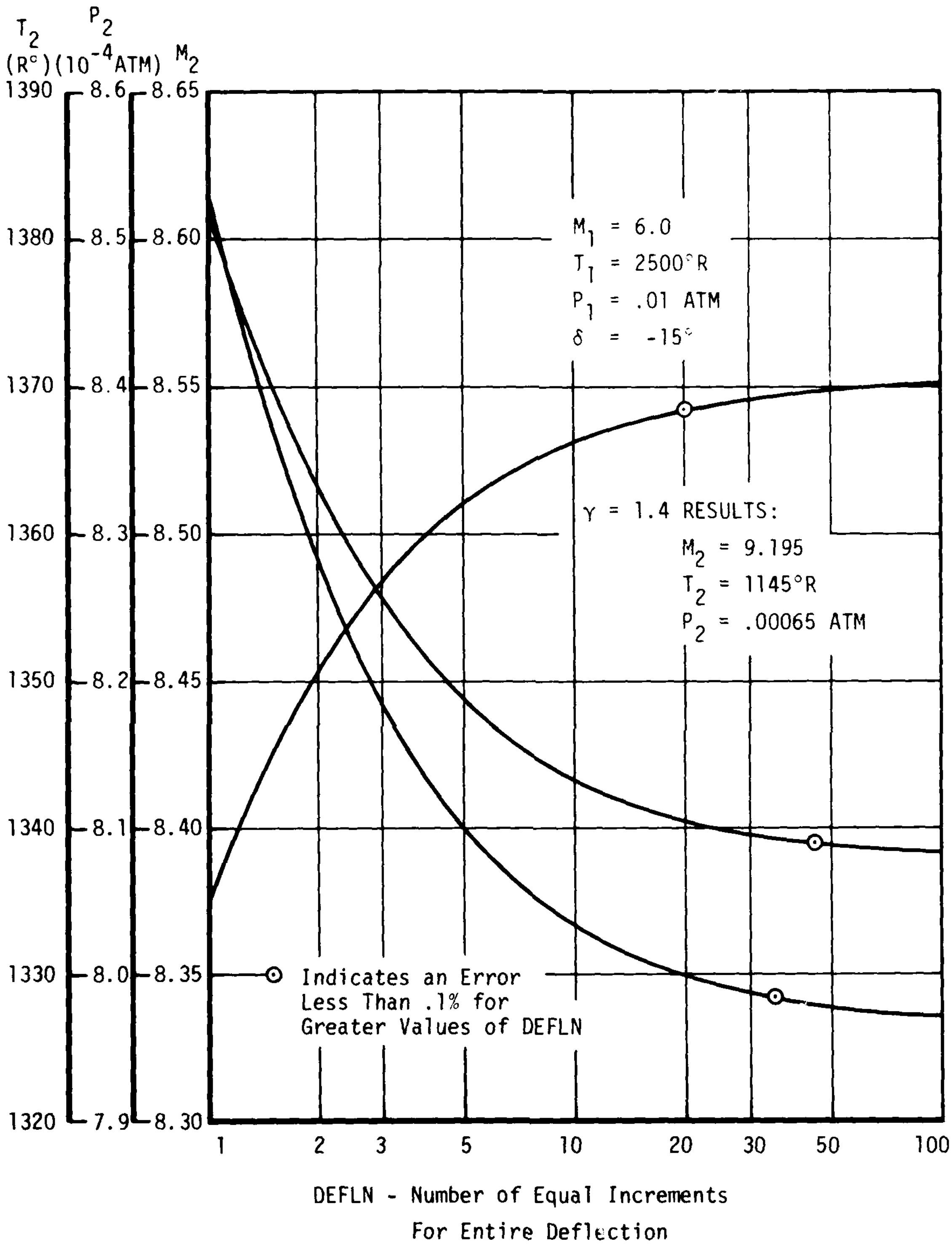


Figure 4. Effect of Calculation Interval on Results Obtained from Subroutine RGEXP

smaller increment would be required for the same degree of precision. Similarly, for lower Mach number cases a larger increment would be appropriate.

The output quantities determined by use of subroutine RGEXP are the downstream conditions T2, P2, and VM2, in the same units as the corresponding input parameters. Also calculated are the upstream and downstream Mach angles (VMU1 and VMU2) in degrees. A listing of subroutine RGEXP is included as Table A-12. Subroutines EXN and THAIR are required for use in conjunction with subroutine RGEXP.

4. OBLIQUE SHOCK WAVE - AXISYMETRIC

The basic Taylor-Maccoll Theory for flow over a cone at zero degrees angle of attack was discussed in Section II-4. All the development and equations through Equation 44 are equally valid for real gas and ideal gas calculations. Equation 45, however, is applicable only to an ideal gas. A different approach will therefore be required for real gas calculations. The starting point (Equation 44) will be repeated here for convenience:

$$\frac{d^2u}{d\theta_r^2} + \frac{du}{d\theta_r} \left(\cot \theta_r + \frac{1}{\rho} \frac{d\rho}{d\theta_r}\right) + 2u = 0 \tag{44}$$

The solution is a continuous function in the region between the surface and the shock, so expansion in a convergent Taylor series is possible:

$$\left(\frac{du}{d\theta_r}\right)_{\theta_r+\Delta\theta_r} = \left(\frac{du}{d\theta_r}\right)_{\theta_r} + \Delta\theta_r \left(\frac{d^2u}{d\theta_r^2}\right)_{\theta_r} \tag{92}$$

Then:

$$(u)_{\theta_r + \Delta\theta_r} = (u)_{\theta_r} + \Delta\theta_r \left[\frac{\left(\frac{du}{d\theta_r}\right)_{\theta_r} + \left(\frac{du}{d\theta_r}\right)_{\theta_r + \Delta\theta_r}}{2} \right]$$

$$(u)_{\theta_r + \Delta\theta_r} = (u)_{\theta_r} + \Delta\theta_r \left[\frac{\left(\frac{du}{d\theta_r}\right)_{\theta_r} + \left(\frac{du}{d\theta_r}\right)_{\theta_r} + \Delta\theta_r \left(\frac{d^2u}{d\theta_r^2}\right)_{\theta_r}}{2} \right]$$

$$(u)_{\theta_r + \Delta\theta_r} = (u)_{\theta_r} + \Delta\theta_r \left(\frac{du}{d\theta_r}\right)_{\theta_r} + \frac{(\Delta\theta_r)^2}{2} \left(\frac{d^2u}{d\theta_r^2}\right)_{\theta_r} \tag{93}$$

Using Equations 42, 44, 92, and 93 the numerical integration procedure, proceding from the cone surface (where all conditions are specified), to the shock wave, is as follows:

Step 1: Set $\theta_{r_o} = \delta$, $u_o = u_{SURF}$, $v_o = 0$ (no flow into or out of surface),

$T_o = T_{SURF}$, and $P_o = P_{SURF}$

Step 2: Find s, h_{SURF}, $\rho_o = f\,(T_{SURF}, P_{SURF})$

Step 3: Find $h_t = h_{SURF} + \dfrac{u^2_{SURF}}{2gJ}$

Step 4: Set $i = 1$

Step 5: Find $\dfrac{d^2u}{d\theta_r^2} = -2u_{i-1} - v_{i-1} \left[\cot \theta_{r_{i-1}} + \dfrac{1}{\rho_{i-1}} \left(\dfrac{d\rho}{d\theta_r}\right)_{i-1} \right]$

Step 6: Find $\theta_{r_i} = \theta_{r_{i-1}} + \Delta\theta_r$

Step 7: Find $v_i = v_{i-1} + \Delta\theta_r \left(\dfrac{dv}{d\theta_r}\right) = v_{i-1} + \Delta\theta_r \left(\dfrac{d^2u}{d\theta_r^2}\right)$

Step 8: Find $u_i = u_{i-1} + \Delta\theta_r (v_{i-1}) + \frac{(\Delta\theta_r)^2}{2} \left(\frac{d^2 u}{d\theta_r^2} \right)$ (From Equation 93)

Step 9: Find $w_i = \sqrt{u_i^2 + v_i^2}$

Step 10: Find $h_i = h_t - \frac{w_i^{\,2}}{2gJ}$

Step 11: Find $T_i, P_i = f\,(h_i, S)$

Step 12: Find $M_{2_i} = w_i \sqrt{g\gamma RT_i}$

Step 13: Find $\phi_i = \tan^{-1}\left(\frac{v_i}{u_i} \right) + \theta_{r_i}$

Step 14: Find $\left(\frac{d\rho}{d\theta_r} \right)_i = \frac{\rho_{i-1} - \rho_i}{\Delta\theta_r}$

Step 15: Find $M_{2_{w_i}}$ for 2-D wedge with $\delta = \phi_i$ and $\theta = \theta_{r_i}$

Step 16: If $M_{2_{w_i}} > M_{2_i}$, set $i = i+1$ and return to Step 5

Step 17: If $M_{2_{w_i}} < M_{2_i}$, find $\theta = \theta_{r_{i-1}} + \Delta\theta_r \frac{\left(M_{2_{w_{i-1}}} - M_{2_{i-1}} \right)}{\left(M_{2_{w_{i-1}}} - M_{2_{w_i}} - M_{2_{w_i}} - M_{2_i} \right)}$

Step 18: Find $M_{2_s} = M_{2_{w_{i-1}}} + \frac{\theta - \theta_{r_{i-1}}}{\theta_{r_i} - \theta_{r_{i-1}}} \left(M_{2_{w_i}} - M_{2_{w_{i-1}}} \right)$

Step 19: Find $\phi_s = \phi_{i-1} + \frac{\theta - \theta_{r_{i-1}}}{\theta_{r_i} - \theta_{r_{i-1}}} (\phi_i - \phi_{i-1})$

Step 20: Find $M_1 = M_{1_{i-1}} + \frac{\theta - \theta_{r_{i-1}}}{\theta_{r_i} - \theta_{r_{i-1}}} \left(M_{1_i} - M_{1_{i-1}} \right)$

A computer program has been written to perform the calculation procedure outlined above. A listing of this program (Program RCONE) is included as Table A-13. The use of subroutine THAIR is required in conjunction with Program RCONE. The input data to this program are: DELT, the cone half angle in degrees; DTHETA, the calculation interval in degrees; USRF, the surface air velocity in feet per second; TSRF, the surface air temperature in °R; and, PSRFA, the surface air pressure in atmospheres. The output for one set of input data is on a single sheet. A sample output is included as Figure 5. The input for this sample case is as follows: DELT = 20., DTHETA = 1., USRF = 2000., TSRF = 1000., and PSRF = .5.

A calculation interval of 1° has been shown to be adequately small for most ideal gas calculations. (See Figure 2.) This calculation interval is generally adequate for real gas calculations as well. If extreme precision is required, a smaller interval can be selected.

The calculation procedure and computer program discussed in the preceding paragraphs is an inverse approach to the problem. The conditions on the cone surface have been specified and the conditions upstream of the shock wave are calculated. In most cases, however, the data available is the conditions upstream of the shock wave, and the information to be determined is the conditions downstream of the shock wave. It is possible to obtain the desired information through iterating with program RCONE. In some cases this may be the most appropriate technique.

REAL GAS CONE FLOW CALCULATION

CONE HALF ANGLE = 20.000DEGREES

USURF = 2000.0FPS

TSURF = 1000.0R   PSURF = 5.000E-01ATM

THETA-DEG	V2-FPS	U2-FPS	W2-FPS	PHI-DEG	M2	T2-R	P2-ATM
21.00	-69.8	1999.4	2000.6	19.00	1.2971	999.[illegible]	4.9965E-01
22.00	-136.5	1997.6	2002.2	18.09	1.2985	999.3	4.9570E-01
23.00	-200.5	1994.7	2004.7	17.26	1.3005	998.5	4.9728E-01
24.00	-262.3	1990.6	2007.8	16.49	1.3032	997.5	4.9547E-01
25.00	-322.2	1985.5	2011.5	15.78	1.3063	996.3	4.9335E-01
26.00	-380.4	1979.4	2015.6	15.12	1.3098	995.0	4.9097E-01
27.00	-437.2	1972.2	2020.1	14.50	1.3137	993.[illegible]	4.8836E-01
28.00	-492.5	1964.1	2025.0	13.92	1.3179	991.5	4.8555E-01
29.00	-547.2	1955.1	2030.2	13.36	1.3223	990.3	4.8257E-01
30.00	-600.7	1945.0	2035.7	12.84	1.3270	988.5	4.7942E-01
31.00	-653.3	1934.1	2041.4	12.34	1.3320	986.5	4.7613E-01
32.00	-705.1	1922.2	2047.5	11.86	1.3372	984.5	4.7266E-01
33.00	-756.2	1909.5	2053.8	11.39	1.3427	982.5	4.6909E-01
34.00	-806.7	1895.8	2060.4	10.95	1.3484	980.4	4.6536E-01
35.00	-856.7	1881.3	2067.2	10.52	1.3544	978.1	4.6147E-01
36.00	-906.2	1865.9	2074.4	10.10	1.3607	975.7	4.5742E-01
37.00	-955.4	1849.7	2081.9	9.68	1.3673	973.2	4.5320E-01
38.00	-1004.3	1832.6	2089.7	9.28	1.3743	970.5	4.4876E-01
39.00	-1052.9	1814.6	2098.0	8.88	1.3816	967.8	4.4415E-01
40.00	-1101.5	1795.8	2106.8	8.48	1.3894	964.5	4.3927E-01
41.00	-1150.1	1776.2	2116.0	8.08	1.3977	961.7	4.3411E-01
42.00	-1198.9	1755.7	2126.0	7.67	1.4067	958.3	4.2861E-01
43.00	-1248.1	1734.3	2136.7	7.26	1.4164	954.5	4.2271E-01
44.00	-1297.8	1712.1	2148.4	6.84	1.4270	950.5	4.1630E-01

M1 = 1.6606   THETA S = 44.262DEG

(P2/P1)SHK = 1.392   (P2/P1)SURF = 1.[illegible]
(T2/T1)SHK = 1.098   (T2/T1)SURF = 1.156

DS/R = .0037

Figure 5. Sample Output for Program RCONE

However, a computer subroutine has been written to perform the iterative computation procedure required with the generally available data. The calling statement for this subroutine is:

```
CALL RGCON (VM1, T1, P1A, DELT, THE, THETAS, DSR, VM2C, PHIC, PRC, TRC,
VM2S, PRSHK, TRSHK, PHIS, VM2SRF, PRSRF, TRSRF)
```

The first five items in this sequence are inputs. VM1 is the upstream Mach number, T1 is the upstream static temperature in °R, P1A is the upstream static pressure in atmospheres, and DELT is the cone half angle in degrees. The fifth input, THE, has been included to provide data on one specific ray lying between the cone surface and the shock wave. THE is the ray angle, in degrees, for which data is required. The parameters found by use of subroutine RGCON are as follows: THETAS is the shock wave angle, DSR is the non-dimensionalized entropy increase across the shock wave, VM2C is the Mach number on the specified ray, PHIC is the flow angle on the specified ray, PRC is the ratio of the static pressure on the specified ray to the upstream static pressure, TRC is the ratio of the static temperature on the specified ray to the upstream static temperature, VM2S is the Mach number just downstream of the shock wave, PRSHK is the static pressure ratio across the shock wave, TRSHK is the static temperature ratio across the shock wave, PHIS is the flow angle just downstream of the shock wave, VM2SRF is the Mach number on the cone surface, PRSRF is the ratio of the static pressure on the surface to the upstream static pressure, and TRSRF is the ratio of the static temperature on the surface to the upstream static temperature. All angles are in degrees. A listing of subroutine RGCON is included as Table A-14. The use of subroutines THAIR and NEWT is required in conjunction with subroutine RGCON.

To illustrate the use of subroutine RGCON, and additional program has been written. The listing of this program, which is entitled program RGCONE, is included as Table A-15. The input data to this program are: VM1, the upstream Mach number; T1, the upstream static temperature in °R; P1A, the upstream static pressure in atmospheres; DELT, the cone half-angle in degrees; and THE, the angle in degrees of a ray for which data is desired. A sample output for program RGCONE is

included as Figure 6. The input data used to generate Figure 6 is as follows: VM1=3.0. T1=390°R, P1A=1.0 atm, DELT=20, THE=22. This data is similar to that used by program CONES with subroutine CONSK to obtain the printout presented in Figure 3 for ideal gas with ratio of specific heats equal 1.4. It can be seen by comparison of Figures 3 and 6 that the results are very much in agreement. This is to be expected for the example shown. Note, however, that the computer time required to generate Figure 6 was in excess of 50 seconds, while the computer time for Figure 3-type calculations is nearly insignificant.

```
            REAL GAS CONE FLOW CALCULATION

                       M1 = 3.000
T1 = 390.0R               P1 = 1.0000E+00ATM
                    DELTA = 20.000DEG

                  THETA S = 2[illegible].522DEG

                     DS/R =    .0573


                AT THETA = 22.0000EG:

                       M2 = 2.2[illegible]6
                      PHI = 13.149DEG
                    P2/P1 = 2.7[illegible]
                    T2/T1 = 1.3632


                   AT SURFACE:

                       M2 = 2.291
                    P2/P1 = 2.7[illegible]
                    T2/T1 = 1.3[illegible]3


                    AT SHOCK:

                       M2 = 2.357
                      PHI = 12.391DEG
                    P2/P1 = 2.3991
                    T2/T1 = 1.3[illegible]
```

Figure 6. Sample Output for Program RGCONE

SECTION IV

EXAMPLES USING DEVELOPED TECHNIQUES AND SUBROUTINES

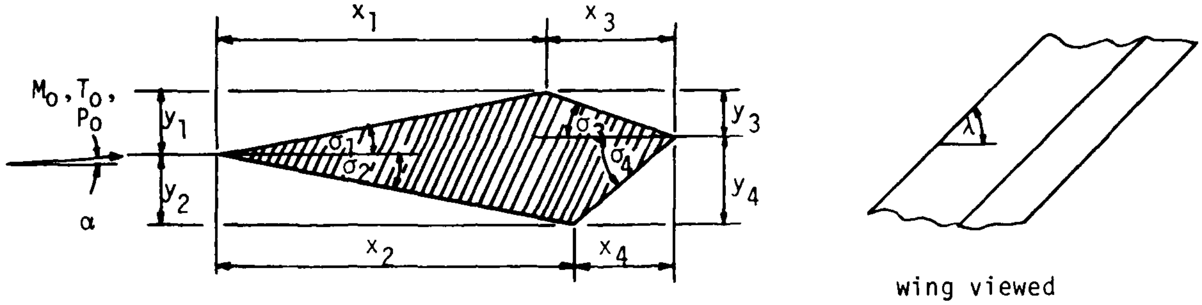
The utility of the various computation techniques and computer subroutines discussed in the preceding sections will be demonstrated in this section. Three typical supersonic airflow problems will be employed to accomplish the demonstration. Each problem will be approached in three ways. First, for a low Mach number case, hand calculations, using the charts in Appendix B, will be described. Second, a computer program, written to use the ideal gas subroutines of Section II, will be described and demonstrated for low, intermediate, and high Mach number cases. Finally, a computer program, written to use the real gas subroutines of Section III, will be described and demonstrated for the same Mach numbers. A comparison of the final two techniques will illustrate the validity of the ideal gas techniques.

Note that all the techniques discussed in this document are for inviscid air flows only. Inclusion of viscous effects and correction techniques is not within the planned scope of this document. In reality, all airflows are dependent, to a greater or lesser degree, on viscous effects. For this reason, any comparisons of results using the techniques discussed in this document with actual test data are not valid.

The first of the supersonic airflow problems to be considered is a two-dimensional swept wing of infinite span. The procedure for determining the lift and drag of such a wing will be discussed. The second problem concerns a twc dimensional supersonic inlet for an airbreathing propulsion device. The back pressure required to maintain the normal shock on the inlet lip,and the total pressure recovery just downstream of the normal shock will be calculated for an inlet with a two-step ramp, at Mach numbers on or below the design Mach number. The third problem is similar to the second, but uses a conical centerbody inlet at zero angle of attack. Again, the back pressure required to keep the normal shock on the inlet lip and the total pressure recovery just downstream of the normal shock will be calculated for cases at or below the design Mach number.

1. PROBLEM 1 - DETERMINATION OF PRESSURE DISTRIBUTION AND LIFT AND DRAG FOR A SWEPT WING OF INFINITE SPAN

The basic features and dimensions of the wing being considered are illustrated in the following diagrams:



wing cross section
parallel to flight direction

wing viewed
from above

The parameters to be specified for this problem are: the upstream Mach number, static temperature, and static pressure, M_0, T_0, and P_0; the wing angle of attack, α; and the wing dimensional quantities, y_1, y_2, σ_1, σ_2, σ_3, σ_4, and λ. The pressure distribution on the wing upper and lower surfaces, and the wing lift and drag are to be calculated.

For the case demonstrating the hand calculation technique, the specifications will be

$$M_o = 2.5,\ T_o = 500°R,\ P_o = .5\ \text{atm},\ \alpha = 2°$$

$$y_1 = .2\ \text{ft},\ y_2 = .5\ \text{ft}$$

$$\sigma_1 = 5°,\ \sigma_2 = 15°,\ \sigma_3 = 20°,\ \sigma_4 = 10°$$

$$\lambda = 60°$$

The first step in the calculation procedure is to determine the equivalent upstream conditions due to the wing sweep. From Figure B-9, we find

$$\frac{M_{e_o}}{M_o} = .867 \rightarrow M_{e_o} = 2.5\ (.867) = 2.168$$

And, from Figure B-10

$$\alpha_e = 2.3°$$

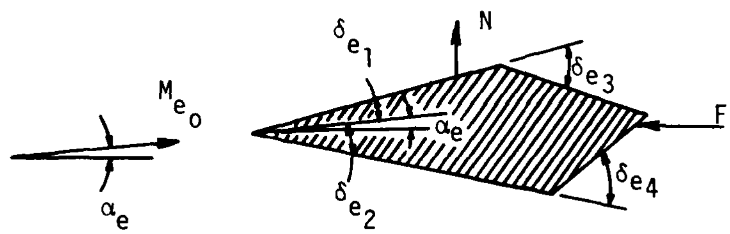
$$\sigma_{e_1} = 5.7°$$

$$\sigma_{e_2} = 17.2°$$

$$\sigma_{e_3} = 22.8°$$

$$\sigma_{e_4} = 11.5°$$

To determine the pressures and forces on the wing surfaces, one must first determine the deflections the airflow will encounter. The deflections and forces are shown in the following diagram:



wing cross section
normal to leading edge

The deflections then are

$$\delta_{e_1} = \sigma_{e_1} - \alpha_e = 5.7° - 2.3° \approx 3.4°$$

$$\delta_{e_2} = \sigma_{e_2} + \alpha_e = 17.2° + 2.3° = 19.5°$$

$$\delta_{e_3} = - \sigma_{e_1} - \sigma_{e_3} = - 5.7° - 22.8° = -28.5°$$

$$\delta_{e_4} = - \sigma_{e_2} - \sigma_{e_4} = - 17.2° - 11.5° = - 28.7°$$

The next step is to determine the unspecified dimensions of the wing. Referring to the diagram at the beginning of Section IV.1:

$$x_1 = \frac{y_1}{\tan \sigma_1} = \frac{.5}{\tan 5°} = 2.286 \text{ ft}$$

$$x_2 = \frac{y_2}{\tan \sigma_2} = \frac{.5}{\tan 15°} = 1.866 \text{ ft}$$

$$x_3 = \frac{y_3}{\tan \sigma_3} \tag{94}$$

$$x_4 = \frac{y_4}{\tan \sigma_4} \tag{95}$$

$$y_3 + y_4 = y_1 + y_2 \tag{96}$$

$$x_1 + x_3 = x_2 + x_4 \tag{97}$$

Equations 94 through 97 can be solved for x_3 to find:

$$x_3 = \frac{y_1 + y_2 + (x_2 - x_1) \tan \sigma_4}{\tan \sigma_3 + \tan \sigma_4}$$

$$x_3 = \frac{.2 + .5 + (1.866 - 2.286) \tan 10°}{\tan 20° + \tan 10°} = 1.158 \text{ ft}$$

Then, using Equations 97, 95, and 96:

$$x_4 = 2.286 + 1.158 - 1.866 = 1.578 \text{ ft}$$

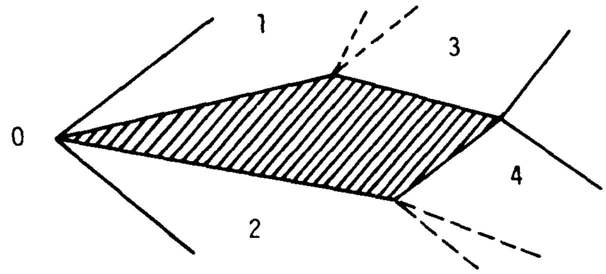
$$y_4 = 1.578 \tan 10° = .278 \text{ ft}$$

$$y_3 = .2 + .5 - .278 = .422 \text{ ft}$$

The total cord of the wing is:

$$x = x_1 + x_3 = 2.286 + 1.158 = 3.444 \text{ ft}$$

To calculate the pressure distribution over the wing, a sketch indicating the approximate location of shock waves and expansion fans will be helpful.



Since we are interested only in the forces on the wing surfaces there is no need to calculate the oblique shock wave angles or the Mach line angles for the expansions. In addition, there is no reason to consider the trailing edge oblique shock waves. The Mach number change and static pressure ratio across each of the leading edge shock waves can be determined by use of Figures B-4 and B-6. However, we should first check to make sure that the deflections are not so great as to preclude an attached shock wave. This is done with Figure B-2. Using Figure B-2, we find that at $M_{e_o} = 2.168$ the maximum deflection is 25.6°. Both of the leading edge deflections are less than 25.6°, so the oblique shock waves will be attached. Then, using Figures B-4 and B-6, we find:

$$\frac{M_1}{M_{e_o}} = .940 \qquad \frac{M_2}{M_{e_o}} = .645$$

$$\frac{P_o}{P_1} = .819 \qquad \frac{P_o}{P_2} = .349$$

And:

$$M_1 = .94\ (2.168) = 2.038$$

$$M_2 = .645\ (2.168) = 1.398$$

$$P_1 = \frac{.5}{.819} = .611 \text{ atm}$$

$$P_2 = \frac{.5}{.349} = 1.433 \text{ atm}$$

The expansions at the top surface and bottom surface corners can be addressed by use of Figure B-11. The Prandtl-Meyer angles upstream of the corners are

$$\nu_1 = 27.3°$$

$$\nu_2 = 8.9°$$

Downstream of the corners,

$$\nu_3 = \nu_1 - \delta_{e_3} = 27.3° + 28.5° = 55.8°$$

$$\nu_4 = \nu_2 - \delta_{e_4} = 8.9° + 28.7° = 37.6°$$

Again using Figure B-11,

$$M_3 = 3.33$$

$$M_4 = 2.44$$

And, since the expansion flow is isentropic,

$$P_3 = P_1 \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_3^2} \right]^{\frac{\gamma}{\gamma-1}}$$

$$P_4 = P_2 \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_4^2} \right]^{\frac{\gamma}{\gamma-1}}$$

For $\gamma = 1.4$:

$$P_3 = .611 \left[\frac{1 + .2\ (2.038)^2}{1 + .2\ (3.33)^2} \right]^{3.5} = .085 \text{ atm.}$$

$$P_4 = 1.433 \left[\frac{1 + .2\ (1.398)^2}{1 + .2\ (2.44)^2} \right]^{3.5} = .292 \text{ atm.}$$

All the forces on the wing surfaces have now been defined. All that remains is to sum these forces in the appropriate orientation. The force normal to the wing as illustrated by the figure on page 52 is defined:

$$N = P_2 x_2 + P_4 x_4 - P_1 x_1 - P_3 x_3 \qquad (98)$$

The force along the wing axis is defined:

$$F = P_3 y_3 + P_4 y_4 - P_1 y_1 - P_2 y_2 \qquad (99)$$

Using Equations 98 and 99,

$$N = 2116[1.433\ (1.866) + .292\ (1.578) - .611\ (2.286) - .085\ (1.158)]$$
$$= 3469.3 \text{ lbs/ft of wing span}$$

$$F = 2116\ [.085\ (.422) + .292\ (.278) - .611\ (.2) - 1.433\ (.5)]$$
$$= -1527.0 \text{ lbs/ft of wing span}$$

The lift and drag can be found from the normal and wing axis drag by using the equations:

$$\text{Lift} = N \cos \alpha + F \sin \alpha \qquad (100)$$

$$\text{Drag} = N \sin \alpha - F \cos \alpha \qquad (101)$$

Then: $\text{Lift} = 3469.3 \cos 2° - 1527.0 \sin 2° = 3413.9$ lbs/ft of wing span

$\text{Drag} = 3469.3 \sin 2° + 1527.0 \cos 2° = 1647.1$ lbs/ft of wing span

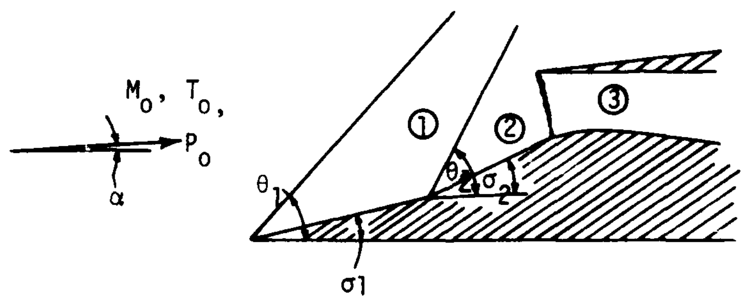
A computer program designed to accomplish the calculations described above has been written. A listing of this program is included as Table A-16. The input for this program is: MO, TO, and PO, the Mach number, static temperature, and static pressure upstream of the wing: Y1 and Y2, the thickness of the wing above and below the centerline; SIG1, SIG2, SIG3, and SIG4, the angle of each wing surface with respect to the wing centerline; LAMBDA, the wing sweep angle; and ALPHA, the wing angle of attack. Temperature is in °R, pressure is in atmospheres, dimensions are in feet, and angles are in degrees. The output of this program is a single sheet

for each input case. Output for three sample problems is included as Figures 7, 8, and 9. The first case (Figure 7) has input data identical to the preceding hand calculated example. The second case as illustrated in Figure 8 is for a wing at a Mach number of 6.0. The final case is for a wing at a Mach number of 12.0. This case is illustrated in Figure 9.

The Mach 6.0 and Mach 12.0 cases illustrated are actually not valid since the calculations have been made on an ideal gas, γ = 1.4, basis. The degree of invalidity will be demonstrated by comparison with results obtained on a real gas basis for identical input data. In order to accomplish this comparison, the swept wing computer program has been modified to include real gas subroutines RGSK and RGEXP. A listing of the revised computer program is included as Table A-17. The same input sets used to generate Figures 7, 8, and 9 have been used with the revised program to generate Figures 10, 11, and 12. Comparison of the corresponding output sheets indicates that for the Mach 2.5 example the results are very similar. At Mach 6.0 there is some difference, and at Mach 12.0 there are major discrepancies.

2. PROBLEM 2 - DETERMINATION OF BACK PRESSURE AND PRESSURE RECOVERY FOR A TWO-DIMENSIONAL INLET

The inlet to be considered is as described in the following sketch:



To avoid undue complication a number of restrictions will be placed on the cases to be addressed. The oblique shock waves will not intersect within the capture area of the inlet, and neither of the oblique shock waves will enter the inlet; the angle of attack will be less than the first ramp angle; neither of the ramp angles will be so large that

```
PRESSURE DISTRIBUTION OVER SWEPT WING
OF INFINITE SPAN

M0= 2.5    P0=5.0[illegible]-01ATM    T0= 500.[illegible]

ALPHA=  2.0DEG

Y1= .20FT    Y2= .50FT

SIG1= 5.0DEG    SIG2=15.0DEG    SIG3=20.0DEG    SIG4=10.0[illegible]

LAMBDA= 60.0DEG

MN0= 2.166

SIGE1= 5.77DEG    SIGE2=17.19DEG    SIGE3=22.80DEG    SIGE4=11.51[illegible]

DELTE1= 3.46DEG  DELTE2=19.50DEG  DELTE3=-28.56DEG  DELTE4=-28.7[illegible]

X1= 2.29FT        X2= 1.97FT        X3= 1.15FT        X4= 1.5[illegible]FT

TOTAL LENGTH= 3.44FT

Y3= .42FT      Y4= .28FT

DELTMAX=25.61DEG

M1= 2.03          M2= 1.40          M3= 3.34          M4= 2.44

T1= 529.9R        T2= 696.9R        T3= 299.9R        T4= 443.[illegible]

P1=6.122E-01ATM  P2=1.426E+00ATM  P3=8.344E-02ATM  P4=2.924E-01ATM

DRAG=  1640.2LBS/FT OF WING SPAN

LIFT=  3385.9LBS/FT OF WING SPAN
```

Figure 7. Mach 2.5 Case for Ideal Gas Swept Wing Problem

PRESSURE DISTRIBUTION OVER SWEPT WING
OF INFINITE SPAN

M0= 6.0 P0=1.00E-01ATM T0= 500.R

ALPHA= 5.00DEG

Y1= .20FT Y2= .30FT

SIG1=15.0DEG SIG2=20.0DEG SIG3= 4.00DEG SIG4= 3.00DEG

LAMBDA= 60.00DEG

ME0= 5.910

SIGE1=15.220DEG SIGE2=20.230DEG SIGE3= 4.060DEG SIGE4= 3.050DEG

DELTE1=10.140DEG DELTE2=25.360DEG DELTE3=-19.280DEG DELTE4=-23.330DEG

X1= .75FT X2= .32FT X3= 4.12FT X4= 4.04FT

TOTAL LENGTH= 4.87FT

Y3= .29FT Y4= .21FT

DELTMAX=42.350EG

M1= 4.57 M2= 2.77 M3= 7.17 M4= 4.21

T1= 770.6R T2=1574.0R T3= 353.2R T4= 677.9R

P1=3.668E-01ATM P2=1.313E+00ATM P3=2.390E-02ATM P4=1.701E-01ATM

DRAG= 1152.0LBS/FT OF WING SPAN

LIFT= 2865.5LBS/FT OF WING SPAN

Figure 8. Mach 6 Case for Ideal Gas Swept Wing Problem

```
        PRESSURE DISTRIBUTION OVER SWEPT WING
                  OF INFINITE SPAN

          M0=12.0    P0=5.00E-02ATM    T0= 300.R

                    ALPHA=  5.00DEG

               Y1= .20FT    Y2= .30FT

 SIG1=15.00DEG   SIG2=20.00DEG   SIG3= 4.00DEG   SIG4= 3.00DEG

                   LAMBDA= 30.00DEG


                     MEC=11.619

 SIGE1=15.22DEG   SIGE2=20.28DEG  SIGE3= 4.06DEG   SIGE4= 3.05DEG

DELTE1=10.14DEG  DELTE2=25.36DEG  DELTE3=-19.28DEG  DELTE4=-23.33DEG

   X1=  .75FT        X2=  .32FT       X3= 4.12FT       X4= 4.04FT

                TOTAL LENGTH= 4.87FT

               Y3= .29FT     Y4= .21FT


                 DELTMAX=44.75DEG

   M1= 7.24          M2= 3.44         M3=14.96          M4= 5.45

  T1=2910.3R        T2=6880.4R       T3= 503.9R        T4=3332.7R

P1=4.673E-01ATM  P2=2.290E+00ATM  P3=3.685E-03ATM  P4=1.811E-01ATM

          DRAG=  1977.6LBS/FT OF WING SPAN

          LIFT=  4616.2LBS/FT OF WING SPAN
```

Figure 9. Mach 12 Case for Ideal Gas Swept Wing Problem

```
PRESSURE DISTRIBUTION OVER [illegible]
OF INFINITE [illegible]

M= 2.5     P=5.00[illegible]-01[illegible]     T= 5[illegible].

ALPHA= 2.0[illegible]

Y1= .20F[illegible]     Y2= .5[illegible]

SIG1= 5.00[illegible]   SIG2=15.[illegible]   SIG3=20.[illegible]   SIG4=10.[illegible]

LAMBDA= 60.00[illegible]

M= 2.166

SIGE1= 5.77DEG   SIGE2=17.1[illegible]   SIGE3=22.[illegible]   SIGE4=11.51[illegible]

DELTE1= 3.46DEG   DELTE2=13.50[illegible]   DELTE3=-20.56[illegible]   DELTE4=-20.7[illegible]

X1= 2.29FT   X2= 1.87[illegible]   X3= 1.15[illegible]   X4= 1.5[illegible]

TOTAL LENGTH= 3.44FT

Y3= .42[illegible]     Y4= .28[illegible]

DELTAX= 0.00[illegible]

M1= 2.03   M2= 1.40   M3= 3.34   M4= 2.43

T1= 529.73   T2= 6[illegible]5.4[illegible]   T3= 2[illegible].[illegible]   T4= 443.1

P1=6.113E-01ATM   P2=1.424 +00ATM   P3=9.342E-02ATM   P4=2.926E-01ATM

DRAG=  1637.2LBS/FT OF WING SPAN

LIFT=  3378.9LBS/FT OF WING SPAN
```

Figure 10. Mach 2.5 Case for Real Gas Swept Wing Problem

```
PRESSURE DISTRIBUTION OVER SWEPT WING
OF INFINITE SPAN

M0= 6.0    P0=1.00E-01ATM    T0= 500.R

ALPHA=  5.0DEG

Y1= .20FT    Y2= .30FT

SIG1=15.0DEG   SIG2=20.0DEG   SIG3= 4.0DEG   SIG4= 3.0DEG

LAMBDA= 80.0DEG

MF0= 5.910

SIGE1=15.22DEG   SIGE2=20.28DEG   SIGE3= 4.05DEG   SIGE4= 3.05DEG

DELTE1=10.14DEG  DELTE2=25.36DEG  DELTE3=-19.26DEG  DELTE4=-23.33DEG

X1=  .75FT        X2=  .82FT        X3= 4.12FT        X4= 4.04FT

TOTAL LENGTH= 4.67FT

Y3= .23FT      Y4= .21FT

DELTMAX=  -1DEG

M1= 4.57          M2= 2.77          M3= 7.16          M4= 4.11

T1= 757.8R        T2=1512.6R        T3= 353.6R        T4= 898.0R

P1=3.660E-01ATM  P2=1.299E+00ATM  P3=2.402E-02ATM  P4=1.802E-01ATM

DRAG=  1143.5LBS/FT OF WING SPAN

LIFT=  2929.5LBS/FT OF WING SPAN
```

Figure 11. Mach 6 Case for Real Gas Swept Wing Problem

```
              PRESSURE DISTRIBUTION OVER SWEPT WING
                        OF INFINITE SPAN
-------------------------------------------------------------------
          M0=12.0     P0=5.00E-03ATM     T0= 300.R

                        ALPHA=  5.0DEG

                    Y1= .29FT     Y2= .39FT
-------------------------------------------------------------------
   SIG1=15.0DEG   SIG2=20.0DEG    SIG3= 4.0DEG    SIG4= 3.0DEG

                        LAMBDA= 80.0DEG
-------------------------------------------------------------------
                          MEN=11.819

 SIGE1=15.22DEG   SIGE2=20.28DEG   SIGE3= 4.06DEG   SIGE4= 3.05DEG

DELTE1=10.14DEG  DELTE2=25.35DEG  DELTE3=-19.28DEG  DELTE4=-23.33DEG
-------------------------------------------------------------------
   X1=   .75FT        X2=   .82FT       X3= 4.12FT       X4= 4.04FT

                    TOTAL LENGTH= 4.87FT

                  Y3= .29FT       Y4= .21FT
-------------------------------------------------------------------
                     DELTMAX=    -IDEG

   M1= 7.29          M2= 3.48          M3=14.12          M4= 4.72

  T1=1863.7R        T2=5130.7R        T3= 535.6R        T4=3450.8R
-------------------------------------------------------------------
P1=4.567E-01ATM   P2=2.163E+00ATM   P3=4.579E-03ATM   P4=2.678E-01ATM

              DRAG=   1909.3LBS/FT OF WING SPAN
-------------------------------------------------------------------
              LIFT=   5156.0LBS/FT OF WING SPAN
```

Figure 12. Mach 12 Case for Real Gas Swept Wing Problem

attached shock waves are precluded; and, the inlet back pressure will be such that the normal shock is maintained exactly at the cowl lip. The assumption of inviscid flow will, of course, also apply to this problem.

The parameters to be specified for this problem are: the upstream Mach number, static temperature, and static pressure, M_o, T_o, and P_o; the inlet angle of attack, α; and the ramp angles, σ_1 and σ_2. The static pressure at station 3 (inlet back pressure), and the inlet total pressure recovery (P_{t_3}/P_{t_o}) are to be calculated.

For the case demonstrating the hand calculation technique, the specifications will be

$$M_o = 2.5,\ T_o = 500°R,\ P_o = .5 \text{ atm}$$

$$\sigma_1 = 10°,\ \sigma_2 = 20°$$

$$\alpha = -5°$$

The flow deflections over the ramp are easily determined as follows:

$$\delta_1 = \sigma_1 - \alpha = 10° + 5° = 15°$$

$$\delta_2 = \sigma_2 - \sigma_1 = 20° - 10° = 10°$$

Using Figures B-3, B-4, B-6, and B-7, we find

$$\frac{\delta_1}{\theta_1} = .406,\ \frac{M_1}{M_o} = .749,\ \frac{P_o}{P_1} = .405,\ \frac{\Delta s_{0-1}}{R} = .0738$$

And

$$\theta_1 = \frac{15°}{.406} = 36.95° - 5° = 31.95°$$

$$M_1 = .749(2.5) = 1.873$$

$$P_1 = \frac{.5}{.405} = 1.235 \text{ atm}$$

Again using the Appendix B figures, we find:

$$\frac{\delta_2}{\theta_2} = .237,\ \frac{M_2}{M_1} = .812,\ \frac{P_1}{P_2} = .597,\ \frac{\Delta s_{1-2}}{R} = .0145$$

And

$$\theta_2 = \frac{10°}{.237} + \sigma_1 = 52.19°$$

$$M_2 = .812\ (1.873) = 1.521$$

$$P_2 = \frac{1.235}{.597} = 2.069 \text{ atm.}$$

Finally, Figure B-1 is used to determine the conditions following the normal shock. From Figure B-1,

$$M_3 = .694,\ \frac{P_2}{P_3} = .395,\ \frac{\Delta s_{2-3}}{R} = .0798$$

And

$$P_3 = \frac{2.069}{.395} = 5.238 \text{ atm.}$$

This is the back pressure (created by combustion of fuel in the engine) that is required to maintain the normal shock wave exactly at the inlet lip. A higher back pressure would force the normal shock forward of the lip and cause spillage of air. A lower back pressure would allow the normal shock to move into the inlet and a complicated pattern of reflected shocks and expansion waves would then precede the normal shock.

The dimensionless entropy increase for the inlet is determined by summing the increases for each of the shock waves.

$$\frac{\Delta s_{0-3}}{R} = \frac{\Delta s_{0-1}}{R} + \frac{\Delta s_{1-2}}{R} + \frac{\Delta s_{2-3}}{R} = .0738 + .0145 + .0798 = .1681$$

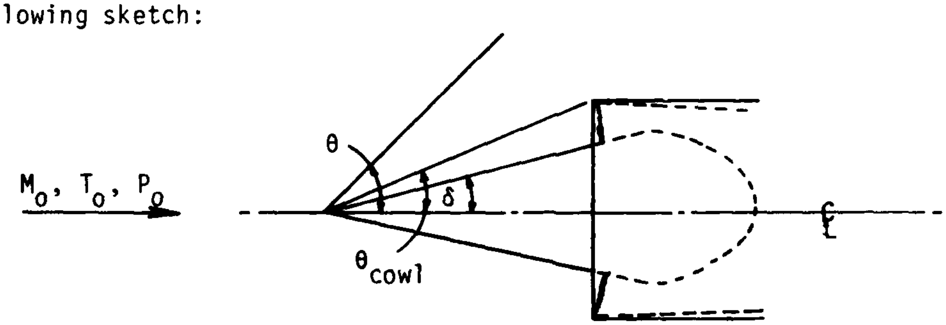
The pressure recovery of the inlet is easily found from the relationship

$$\frac{P_{t_3}}{P_{t_0}} = e^{-\frac{\Delta s_{0-3}}{R}} = e^{-.1681} = .8453$$

A computer program has been written to accomplish the calculations described above. A listing of this program is included as Table A-18. The inputs for this program are MO, TO, PO, the Mach number, static temperature in °R, and static pressure in atmospheres, upstream of the inlet; SIG1 and SIG2, the two inlet ramp angles, in degrees; and, ALPHA, the inlet angle of attack in degrees. The output of this program is on a single sheet, and includes both the ideal gas and the real gas solutions appropriate to the input data. Output for three sample cases is included as Figures 13, 14, and 15. The first case (Figure 13) has input data identical to the preceding hand calculated example. The second case, as illustrated in Figure 14, is for an inlet at a Mach number of 6.0. The third case is for an inlet at Mach 12.0. The output sheet for this case is Figure 15. The degree of discrepancy between real gas and ideal gas calculations can be observed at each of the three Mach numbers.

3. PROBLEM 3 - DETERMINATION OF BACK PRESSURE AND PRESSURE RECOVERY FOR A CONICAL INLET

The inlet to be considered for this problem is described in the following sketch:



The restrictions on this problem include the following:

(1) The inlet is at a zero angle of attack.

(2) The shock wave angle (θ) is greater than the cowl angle (θ_{COWL}).

(3) The cone half angle (δ) is small enough that an attached shock wave is possible.

(4) The back pressure is regulated so that the normal shock wave is exactly on the cowl lip.

BACK PRESSURE AND PRESSURE RECOVERY

FOR 2-D INLET

M0= 2.5 P0=5.00E-01ATM T0= 500.0R

ALPHA=-5.0DEG

SIG1=10.00DEG SIG2=20.00DEG

DELT1=15.00DEG DELT2=10.00DEG

IDEAL GAS SOLUTION

M1= 1.87	P1=1.234E+00ATM	T1= 661.0R
M2= 1.52	P2=2.068E+00ATM	T2= 763.1R
M3= .69	P3=5.235E+00ATM	T3= 1026.1R

THETA1=31.95DEG THETA2=52.13DEG

REQUIRED BACK PRESSURE=5.235E+00ATM

PRESSURE RECOVERY(PT3/PT0)=.8454

REAL GAS SOLUTION

M1= 1.87	P1=1.233E+00ATM	T1= 660.0R
M2= 1.52	P2=2.061E+00ATM	T2= 765.7R
M3= .69	P3=5.240E+00ATM	T3= 1016.5R

THETA1=31.91DEG THETA2=52.00DEG

REQUIRED BACK PRESSURE=5.240E+00ATM

PRESSURE RECOVERY(PT3/PT0)=.8476

Figure 13. Mach 2.5 Case for 2-D Inlet Problem

```
          BACK PRESSURE AND PRESSURE RECOVERY

                    FOR 2-D INLET

   M0= 6.0      P0=5.00E-02ATM      T0= 390.0R

                  ALPHA=-2.0DEG

        SIG1=15.00DEG        SIG2=18.00DEG

       DELT1=17.00DEG       DELT2= 3.00DEG


                 IDEAL GAS SOLUTION

   M1= 3.74     P1=3.627E-01ATM     T1=  842.1R
   M2= 3.53     P2=4.797E-01ATM     T2=  912.7R
   M3=  .45     P3=6.895E+00ATM     T3= 3063.6R

       THETA1=22.85DEG      THETA2=32.59DEG

      REQUIRED BACK PRESSURE=6.895E+00ATM

       PRESSURE RECOVERY(PT3/PT0)=.1004


                  REAL GAS SOLUTION

   M1= 3.74     P1=3.621E-01ATM     T1=  838.1R
   M2= 3.53     P2=4.764E-01ATM     T2=  904.4R
   M3=  .44     P3=6.984E+00ATM     T3= 2797.6R

       THETA1=22.81DEG      THETA2=32.49DEG

      REQUIRED BACK PRESSURE=6.984E+00ATM

       PRESSURE RECOVERY(PT3/PT0)=.0832
```

Figure 14. Mach 6 Case for 2-D Inlet Problem

Figure 15. Mach 12 Case for 2-D Inlet Problem

(5) The properties of the air upstream of the normal shock can be adequately represented by the conditions on a ray midway between the cone surface and the cowl.

(6) The airflow is inviscid.

The specified data for this problem are: the upstream Mach number, static temperature, and static pressure, M_o, T_o, and P_o; the cone half angle, δ; and, the cowl angle, θ_{COWL}. The static pressure at station 2 (inlet back pressure) and the inlet total pressure recovery (P_{t_2}/P_{t_o}) are to be calculated.

For the case demonstrating the hand calculation technique, the specifications will be

$$M_o = 2.5,\ T_o = 500°R,\ P_o = .5 \text{ atm}$$

$$\delta = 15°,\ \theta_{COWL} = 25°$$

The shock wave angle is found by use of Figure B-13 as follows:

$$\frac{\delta}{\theta} = .527 \rightarrow \theta = \frac{15}{.527} = 28.46°$$

Using Figure B-14, we find

$$\left(\frac{M_1}{M_o}\right)_{\theta_r=15°} = .847 \rightarrow M_{1_{\theta_r=15°}} = 2.118$$

And

$$\left(\frac{M_1}{M_o}\right)_{\theta_r=28.46°} = .896 \rightarrow M_{1_{\theta_r=28.46°}} = 2.240$$

Since we cannot find the properties on specific rays between the cone surface and the shock wave by use of the available figures, the further assumption will be made for the hand calculations that the properties vary linearly from the cone surface to the shock wave. The Mach number

on the ray midway between the surface and the cowl ($\theta_{r_{MP}}=20°$) is then found as follows:

$$M_{1_{\theta_{r_{MP}}}} = M_{1_{\theta_r=15°}} + \frac{\theta_{r_{MP}} - \delta}{\theta - \delta}\left(M_{1_{\theta_r=28.46°}} - M_{1_{\theta_r=15°}}\right)$$

$$M_{1_{\theta_{r_{MP}}}} = 2.118 + \frac{20-15}{28.46-15}\ (2.240-2.118) = 2.163$$

To find the static pressure downstream of the shock wave, we use Figure B-16:

$$\left(\frac{P_o}{P_1}\right)_{\theta_r=15°} = .555 \rightarrow P_{1_{\theta_r=15°}} = \frac{.5}{.555} = .9009 \text{ atm.}$$

$$\left(\frac{P_o}{P_1}\right)_{\theta_r=28.46°} = .673 \rightarrow P_{1_{\theta_r=28.46°}} = \frac{.5}{.673} = .7429 \text{ atm.}$$

Again using the linear variation assumption:

$$P_{1_{\theta_{r_{MP}}}} = .9009 + \frac{5}{13.46}\ (.7429 - .9009) = .8422 \text{ atm.}$$

The dimensionless entropy increase is found using Figure B-17:

$$\frac{\Delta s_{0-1}}{R} = .007$$

Using properties on the ray midway between the surface and the cowl as upstream conditions, the conditions downstream of the normal shock are found using Figure B-1:

$$M_2 = .552$$

$$\frac{P_{1_{\theta_{r_{MP}}}}}{P_2} = .1894 \rightarrow P_2 = \frac{.8422}{.1894} = 4.447 \text{ atm.}$$

$$\frac{\Delta s_{1-2}}{R} = .4370$$

The value of P_2 determined above is the back pressure required to maintain the normal shock wave exactly at the inlet lip.

The overall entropy increase is found by summing the losses through the conical shock wave and the normal shock:

$$\frac{\Delta s_{0-2}}{R} = \frac{\Delta s_{0-1}}{R} + \frac{\Delta s_{1-2}}{R} = .007 + .437 = .444$$

The pressure recovery then is:

$$\frac{P_{t_2}}{P_{t_0}} = e^{-.444} = .6415$$

A computer program has been written to accomplish the calculations described above. The only deviation from the illustrated procedure occu because it is possible, using the cone subroutines, to find the conditio on the specific ray that falls midway between the cone surface and the cowl. A listing of the computer program is included as Table A-19. The required input data for this program are M0, T0, and P0, the conditions upstream of the inlet; DELT, the cone half angle in degrees; and CWLANG the cowl angle in degrees. The output of the program is a single sheet and includes both the ideal gas and the real gas solutions appropriate to the input data. Output for three sample cases is included as Figures 16, 17, and 18. The input data for the first case (Figure 16) is identi to the conditions for the hand calculated example. The second case, as illustrated in Figure 17, is for an inlet at a Mach number of 6.0. The third case (Figure 18) is for an inlet at Mach 12.0.

```
         BACK PRESSURE AND PRESSURE RECOVERY

                 FOR CONICAL INLET

   M0= 2.5      P0=5.00E-01ATM      T0= 530.0R

                  DELTA=15.0DEG

                COWL ANGLE=25.00DEG


                 IDEAL GAS SOLUTION

  M1= 2.14      P1=8.737E-01ATM     T1=   587.5R
  M2=  .56      P2=4.517E+00ATM     T2=  1050.6R

                  THETAS=28.45DEG

       REQUIRED BACK PRESSURE=4.517E+00ATM

         PRESSURE RECOVERY(PT2/PT0)=.6521


                  REAL GAS SOLUTION

  M1= 2.14      P1=8.737E-01ATM     T1=   587.5R
  M2=  .56      P2=4.534E+00ATM     T2=  1051.6R

                  THETAS=28.48DEG

       REQUIRED BACK PRESSURE=4.534E+00ATM

         PRESSURE RECOVERY(PT2/PT0)=.6482
```

Figure 16. Mach 2.5 Case for Conical Inlet Problem

```
BACK PRESSURE AND PRESSURE RECOVERY
FOR CONICAL INLET

M0= 6.0      P0=5.[illegible]E-02ATM      T0= 390.0R
DELTA=15.0DEG

COWL ANGLE=15.00DEG

IDEAL GAS SOLUTION

M1= 4.38     P1=2.355E-01ATM      T1=   661.2R
M2=  .43     P2=5.232E+00ATM      T2=  3086.0R

THETAS=19.01DEG

REQUIRED BACK PRESSURE=5.232E+00ATM

PRESSURE RECOVERY(PT2/PT0)=.0751

REAL GAS SOLUTION

M1= 4.38     P1=2.355E-01ATM      T1=   661.2R
M2=  .41     P2=5.350E+00ATM      T2=  2043.4R

THETAS=19.01DEG

REQUIRED BACK PRESSURE=5.350E+00ATM

PRESSURE RECOVERY(PT2/PT0)=.0638
```

Figure 17. Mach 6 Case for Conical Inlet Problem

```
BACK PRESSURE AND PRESSURE RECOVERY

FOR CONICAL INLET

M0=12.0      P0=5.00E-03ATM      T0= 500.0K

DELTA=10.00DEG

COWL ANGLE=11.00DEG


IDEAL GAS SOLUTION

M1= 7.99     P1=3.784E-02ATM     T1= 1081.1K
M2=  .39     P2=2.615E+00ATM     T2=14454.9K

THETAS=11.90DEG

REQUIRED BACK PRESSURE=2.615E+00ATM

PRESSURE RECOVERY(PT2/PT0)=.0043


REAL GAS SOLUTION

M1= 7.99     P1=3.784E-02ATM     T1= 1081.1K
M2=  .32     P2=3.020E+00ATM     T2= 7718.6K

THETAS=11.88DEG

REQUIRED BACK PRESSURE=3.020E+00ATM

PRESSURE RECOVERY(PT2/PT0)=.0013
```

Figure 18. Mach 12 Case for Conical Inlet Problem

APPENDIX A

COMPUTER PROGRAM LISTINGS

Listings of computer programs and subroutines referred to in the text are included in this appendix.

TABLE A-1

SUBROUTINE FOR NORMAL SHOCK CALCULATIONS

(IDEAL GAS)

```
      SUBROUTINE NORM (VM1,T1,P1,VM2,T2,P2,GAM)
      VM2 = SQRT(((GAM-1.)*VM1**2+2.)/(2.*GAM*VM1**2-(GAM-1.)))
      T2 = T1*(2.*GAM*VM1**2-GAM+1.)*((GAM-1.)*VM1**2+2.)/((GAM+1.)*VM1)
     1**2
      P2 = P1*(2.*GAM*VM1**2-GAM+1.)/(GAM+1.)
      RETURN
      END
```

TABLE A-2

SUBROUTINE FOR OBLIQUE SHOCK CALCULATIONS

(IDEAL GAS)

```
      SUBROUTINE OSSK (DELT,VM1,T1,P1,GAM,SW,DELTM,THETA,VM2,T2,P2)
      DEL = DELT*.0174533
      X = 1.-(VM1**2)
      Y = (VM1**2)*(GAM-1.)+2.
      Z = (VM1**2)*(GAM+1.)+2.
      BB = 27.*(Z**2)/(432.*X)+405.*Y*Z/(432.*(X**2))-729.*(Y**2)/(432.*
     1(X**3))
      BC = 27.*Y*(Z**3)/(432.*(X**3))
      COTDM = SQRT((BB+SQRT(BB**2-4.*BC))/2.)
      DELTM = ATAN(1./COTDM)/.0174533
      IF (DELT-DELTM) 1,2,2
    2 THETA = 0.
      VM2 = 0.
      T2 = .
      P2 = .
      RETURN
    1 IF (DELT-.001) 3,3,4
    3 THETA = ATAN(1./SQRT(VM1**2-1.))/.0174533
      VM2 = VM1
      T2 = T1
      P2 = P1
      RETURN
      COTD = COS(DEL)/SIN(DEL)
      P = 2.*COTD*X/Y
      Q = Z/Y
      R = 2.*COTD/Y
      A = (3.*Q-P**2)/3.
      B = (2.*(P**3)-9.*P*Q+27.*R)/27.
      IF (B) 5,5,6
      COPH = SQRT(-27.*(B**2)/(4.*(A**3)))
      PH = ATAN((SQRT(1.-COPH**2))/COPH)
      GO TO 7
    6 COPH = -SQRT(-27.*(B**2)/(4.*(A**3)))
      PH = 3.14159-ATAN((SQRT(1.-COPH**2))/(-COPH))
    7 IF (Q-.001) 8,8,9
    8 EX = 2.*SQRT(-A/3.)*COS(4.1888+(PH/3.))
      GO TO 10
    9 EX = 2.*SQRT(-A/3.)*COS(PH/3.)
   10 THET = ATAN(EX-P/3.)
      THETA = THET/.0174533
      TMS = (VM1*SIN(THET))**2
      VM2 = SQRT(((GAM-1.)*TMS+2.)/(((SIN(THET-DEL))**2)*(2.*GAM*TMS-GAM
     1+1.)))
      T2 = T1*((2.*GAM*TMS-GAM+1.)*((GAM-1.)*TMS+2.))/(((GAM+1.)**2)*TMS)
      P2 = P1*((2.*GAM*TMS-GAM+1.)/(GAM+1.))
      RETURN
      END
```

TABLE A-3

SUBROUTINE FOR SWEPT-WING CONVERSIONS

```
      SUBROUTINE SWEP(ALAM,VM,ALPHA,SIG,VMEQ,ALPEQ,SIGEQ)
      RD=.0174533
      VMU=ASIN(1./VM)/RD
      IF(VMU.GT.ALAM)GO TO 1
      VMEQ=VM*SQRT(1.-(COS(ALPHA*RD)*COS(ALAM*RD))**2)
      ALPEQ=ATAN(SIN(ALPHA*RD)/(SIN(ALAM*RD)*COS(ALPHA*RD)))/RD
      SIGEQ=ATAN(SIN(SIG*RD)/(SIN(ALAM*RD)*COS(SIG*RD)))/RD
      GO TO 2
1     WRITE(6,3)VMU,ALAM
3     FORMAT(1H ,*MACH ANGLE GREATER THAN SWEEP ANGLE (MU=*,1PE10.2,5X,*
     1LAMBDA=*,E10.2,*)*)
2     RETURN
      END
```

TABLE A-4

SUBROUTINE FOR ISENTROPIC TURN CALCULATIONS

(IDEAL GAS)

```
SUBROUTINE EXN (DELT,VM1,T1,P1,GAM,VMU,VM2,T2,P2)
EXTERNAL EXNC
COMMON/EXNP/G1,VNU2
RD=.0174533
DEL = DELT*RD
G1 = SQRT((GAM+1.)/(GAM-1.))
VMU=ASIN(1./VM1)/RD
VNU1 = G1*ATAN(SQRT(VM1**2-1.)/G1)-ATAN(SQRT(VM1**2-1.))
VNU2 = VNU1-DEL
VM2S=VM1+.2
VMAX=1000.
VMIN=1.
CONV=1.E-6
DELTA=.01
CALL NEWT1(VM2,VM2S,VMIN,VMAX,CONV,50,DELTA,4HEXNC,EXNC,X)
T2=T1*(1.+(GAM-1.)*(VM1**2)/2.)/(1.+(GAM-1.)*(VM2**2)/2.)
P2 = P1*(T2/T1)**(GAM/(GAM-1.))
RETURN
END
```

```
FUNCTION EXNC(VM2,DUM1,DUM2,NT)
COMMON/EXNP/G1,VNU2
VNU2C=G1*ATAN(SQRT(VM2**2-1.)/G1)-ATAN(SQRT(VM2**2-1.))
EXNC=VNU2C-VNU2
RETURN
END
```

TABLE A-5

SUBROUTINE FOR FACILITATING ITERATION PROCEDURES

```
      SUBROUTINE NEWT1 (ROOT,X1,A,B,CONVRG,NTIMES,DELTA,FNAME,
     A    FUNCTN,STATUS)
      LOGICAL STATUS
      EXTERNAL FUNCTN
      ITRY = 0
      ITER8 = 1
      XI = X1
 5    FI = FUNCTN(XI,SLOPE,DELTA,ITER8)
      IF(ABS(FI).GT.CONVRG) GO TO 10
 6    ROOT = XI
      FI = FUNCTN(XI,SLOPE,DELTA,ITER8)
      STATUS = .TRUE.
      GO TO 50
 10   IF(DELTA.GT.90.0) GO TO 15
      DEL = DELTA
      XIPD = XI+DEL
      IF(XIPD.LT.A.OR.XIPD.GT.B) XIPD = XI-DEL
      FIPD = FUNCTN(XIPD,SLOPE,DELTA,ITER8)
      SLOPE = (FIPD-FI)/(XIPD-XI)
 15   IF(SLOPE.EQ.0.0) GO TO 26
      XI1 = XI - FI/SLOPE
      IF(ABS(XI-XI1).LE.1.E-05) GO TO 6
      IF(XI1.GE.A) GO TO 17
      XI1 = (A+XI)/2.0
      GO TO 20
 17   IF(XI1.LE.B) GO TO 25
      XI1 = (B+XI)/2.0
 20   IF(ITER8.GT.3) GO TO 25
 25   XI = XI1
      ITER8 = ITER8 + 1
      IF(ITER8.LE.NTIMES) GO TO 5
 26   IF(ITRY.NE.0) GO TO 30
      X = A
      DEL = (B-A)/20.
      FS = FUNCTN(X,SLOPE,DELTA,1)
      SFS = SIGN(1.0,FS)
      XS = X
      DO 27 I=2,21
      X = X + DEL
      F = FUNCTN(X,SLOPE,DELTA,I)
      SF = SIGN(1.0,F)
      IF(ABS(SF-SFS).GT.1.0) GO TO 28
      XS = X
      FS = F
 27   SFS = SF
      GO TO 30
 28   XI = (XS+X)/2.
      ITER8 = 2
      ITRY = 1
```

TABLE A-5 (CONCLUDED)

```
      GO TO 5
30    X = A
      DEL = (B-A)/20.0
      PRINT 35,FNAME
35    FORMAT(1H0,A10,15HDID NOT CONVERG)
      PRINT 36,ITERS,XI,XI1,X1,A,B
36    FORMAT(1H ,6HITERS = ,I2,5X,5HXI = ,E13.6,5X,
     1  6HXI1 = ,E13.6/1H ,9HXFIRST = ,E13.6,5X,
     3  4HA = ,E13.6,5X,4HB = ,E13.6)
      DO 45 I=1,21
      F = FUNCIN(X,SLOPE,DELTA,I)
      PRINT 40,I,X,F
40    FORMAT(1H ,5X,4HI = ,I2,5X,4HX = ,E15.6,5X,4HF = ,E15.6)
45    X = X + DEL
      ROOT = 0.0
      STATUS = .FALSE.
50    RETURN
      END
```

TABLE A-6

PROGRAM FOR CONE FLOW CALCULATIONS

(IDEAL GAS)

```
      PROGRAM CONE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION THETA(200),VC(200),UC(200),WC(200),PHI(200),VM2(200),WM2
     1(200)
      RD=57.29577951
 1    READ (5,*)DELT,USC,GAM,DTHETA
      IF(EOF(5)) 9,10
 10   GF=(GAM-1.)/2.
      WRITE (6,2)GAM,DELT,USC
 2    FORMAT(1H1,55X,*CONE FLOW CALCULATION*,//,53X,*(GAMMA =*,F6.3,*)*,
     1///,51X,*CONE HALF ANGLE =*,F6.2,*DEGREES*,//,53X,*USUPF/C =*,F6.4
     2,///,41X,*THETA-DEG*,4X,*V/C*,6X,*U/C*,5X,*W/C*,4X,*PHI-DEG*,5X,*M
     32*,/)
      THETA(0)=DELT
      VC(0)=0.
      UC(0)=USC
      DO 3 I=1,200
      UCF=1.-(UC(I-1))**2
      DERIV=((GAM-1.)*UCF*UC(I-1)+(GF*UCF*VC(I-1)/TAN(THETA(I-1)/RD))-GA
     1M*UC(I-1)*(VC(I-1)**2)-GF*(VC(I-1)**3)/TAN(THETA(I-1)/RD))/(((GAM+
     21.)*(VC(I-1)**2)/2.)-GF*UCF)
      THETA(I)=THETA(I-1)+DTHETA
      VC(I)=VC(I-1)+DTHETA*DERIV/RD
      UC(I)=UC(I-1)+DTHETA*VC(I-1)/RD+(DTHETA**2)*DERIV/(2.*(RD**2))
      WC(I)=SQRT((UC(I)**2)+(VC(I)**2))
      VM2(I)=SQRT(WC(I)**2/(GF*(1.-WC(I)**2)))
      PHI(I)=ATAN(VC(I)/UC(I))*RD+THETA(I)
      WM1STT=(2.*(TAN(PHI(I)/RD)+(1./TAN(THETA(I)/RD)))/(SIN(2.*THETA(I)
     1/RD)-TAN(PHI(I)/RD)*(GAM+COS(2.*THETA(I)/RD))))*(SIN(THETA(I)/RD)*
     2*2)
      WM2(I)=SQRT((((GAM+1.)**2)*(WM1STT**2)/((SIN(THETA(I)/RD))**2)-4.*
     1(WM1STT-1.)*(GAM*WM1STT+1.))/((2.*GAM*WM1STT-GAM+1.)*((GAM-1.)*WM1
     2STT+2.)))
      IF(WM2(I)-VM2(I)) 4,4,5
 5    WRITE (6,6)THETA(I),VC(I),UC(I),WC(I),PHI(I),VM2(I)
 6    FORMAT(1H ,F47.2,F11.4,2F8.4,F9.2,F10.3)
 3    CONTINUE
 4    THETAS=THETA(I-1)+DTHETA*(WM2(I-1)-VM2(I-1))/((WM2(I-1)-VM2(I-1))-
     1(WM2(I)-VM2(I)))
      VM2S=WM2(I-1)+(THETAS-THETA(I-1))*(WM2(I)-WM2(I-1))/(THETA(I)-THET
     1A(I-1))
      PHIS=PHI(I-1)+(THETAS-THETA(I-1))*(PHI(I)-PHI(I-1))/(THETA(I)-THET
     1A(I-1))
      SA=(SIN((THETAS-PHIS)/RD))**2
      SB=(SIN(THETAS/RD))**2
      SC=VM2S**2
      VM1=SQRT((2.+(GAM-1.)*SC*SA)/(SB*(2.*GAM*SC*SA-GAM+1.)))
      WRITE (6,7)VM1,THETAS
 7    FORMAT(1H0,49X,*M1 =*,F7.4,5X,*THETA S =*,F7.3,*DEG*)
```

TABLE A-6 (CONCLUDED)

```
      VM2SF=SQRT(USC**2/(GF*(1.-(USC**2))))
      PRSHK=(2.*GAM*(VM1**2)*SB-GAM+1.)/(GAM+1.)
      TRSHK=PRSHK*((GAM-1.)*(VM1**2)*SB+2.)/((GAM+1.)*(VM1**2)*SB)
      DSR=(GAM/(GAM-1.))*ALOG(TRSHK)-ALOG(PRSHK)
      VMF=(1.+GF*SC)/(1.+GF*(VM2SF**2))
      PRSRF=PRSHK*(VMF**(GAM/(GAM-1.)))
      TRSRF=TRSHK*VMF
      WRITE (6,8)PRSHK,PRSRF,TRSHK,TRSRF,DSR
 8    FORMAT(1H0,41X,*(P2/P1)SHK =*,F7.3,13X,*(P2/P1)SURF =*,F7.3,/,
     142X,*(T2/T1)SHK =*,F7.3,13X,*(T2/T1)SURF =*,F7.3,//,61X,*DS/R =*,F
     28.4)
      GO TO 1
 9    STOP
      END
```

TABLE A-7

SUBROUTINE FOR CONICAL SHOCK CALCULATIONS

(IDEAL GAS)

```
 SUBROUTINE CONSK(VM1S,DELTS,GAMS,THES,THETASS,DSRS,VM2CS,PHICS,PRC
1S,TRCS,VM2SS,PRSHKS,TRSHKS,PHISS,VM2SRFS,PRSRFS,TRSRFS)
 EXTERNAL CNCLC
 COMMON THE,DELT,GAM,VM1,THETAS,DSR,VM2C,PHIC,PRC,TRC,GF,RD,VM2S,PR
1SHK,TRSHK,PHIS,VM2SRF,PRSRF,TRSRF
 VM1=VM1S
 DELT=DELTS
 GAM=GAMS
 THE=THES
 RD=57.29577951
 GF=(GAM-1.)/2.
 VMG=VM1*(1.-(DELT/200.)-((DELT**2)/5000.))
 USCG=.9*SQRT(GF*(VMG**2)/(1.+GF*(VMG**2)))
 VMNS=(.996-.009*DELT)**2
 USCMN=SQRT(GF*VMNS/(1.+GF*VMNS))
 USCMX=SQRT(GF*(VM1**2)/(1.+GF*(VM1**2)))
 CALL NEWTT(USC,USCG,USCMN,USCMX,.00010,50,.001,5HCNCLC,CNCLC,X)
 THETASS=THETAS
 DSRS=DSR
 VM2CS=VM2C
 PHICS=PHIC
 PRCS=PRC
 TRCS=TRC
 VM2SS=VM2S
 PRSHKS=PRSHK
 TRSHKS=TRSHK
 PHISS=PHIS
 VM2SRFS=VM2SRF
 PRSRFS=PRSRF
 TRSRFS=TRSRF
 RETURN
 END


 FUNCTION CNCLC(USC,DUM1,DUM2,NT)
 COMMON THE,DELT,GAM,VM1,THETAS,DSR,VM2C,PHIC,PRC,TRC,GF,RD,VM2S,PR
1SHK,TRSHK,PHIS,VM2SRF,PRSRF,TRSRF
 DIMENSION THETA(900),VC(900),UC(900),WC(900),PHI(900),VM2(900),WM2
1(900)
 THETA(0)=DELT
 VC(0)=0.
 UC(0)=USC
 DTHETA=.1
 N=0
 DO 3 I=1,900
 UCF=1.-(UC(I-1))**2
```

TABLE A-7 (CONCLUDED)

```
      DERIV=((GAM-1.)*UDF*UC(I-1)+(GF*UDF*VC(I-1)/TAN(THETA(I-1)/R
     1M*UC(I-1)*(VC(I-1)**2)-GF*(VC(I-1)**3)/TAN(THETA(I-1)/RD))/(
     21.)*(VC(I-1)**2)/2.)-GF*UCF)
      THETA(I)=THETA(I-1)+DTHETA
      IF(N)5,10,5
10    IF(THE-THETA(I)) 5,5,6
6     N=I
5     VC(I)=VC(I-1)+DTHETA*DERIV/RD
      UC(I)=UC(I-1)+DTHETA*VC(I-1)/RD+(DTHETA**2)*DERIV/(2.*(RD**2
      WC(I)=SQRT((UC(I)**2)+(VC(I)**2))
      VM2(I)=SQRT(WC(I)**2/(GF*(1.-WC(I)**2)))
      IF(I-1) 8,8,7
8     IF(VM2(I)-VM1) 7,9,9
9     WRITE(6,4) VM1,VM2(I)
4     FORMAT(1H ,10X,*WITH M1 =*,F6.3,* MSURF =*,F6.3,* IS IMPOSSI
      GO TO 2
7     PHI(I)=ATAN(VC(I)/UC(I))*RD+THETA(I)
      WM1STT=(2.*(TAN(PHI(I)/RD)+(1./TAN(THETA(I)/RD)))/(SIN(2.*TH
     1/RD)-TAN(PHI(I)/RD)*(GAM+COS(2.*THETA(I)/RD))))*(SIN(THETA(I
     2**2)
      WM2(I)=SQRT((((GAM+1.)**2)*(WM1STT**2)/((SIN(THETA(I)/RD))**
     1(WM1STT-1.)*(GAM*WM1STT+1.))/((2.*GAM*WM1STT-GAM+1.)*((GAM-1
     2STT+2.)))
      IF(WM2(I)-VM2(I)) 4,4,3
3     CONTINUE
4     THETAS=THETA(I-1)+DTHETA*(WM2(I-1)-VM2(I-1))/((WM2(I-1)-VM2(
     1(WM2(I)-VM2(I)))
      VM2S=WM2(I-1)+(THETAS-THETA(I-1))*(WM2(I)-WM2(I-1))/(THETA(I
     1A(I-1))
      PHIS=PHI(I-1)+(THETAS-THETA(I-1))*(PHI(I)-PHI(I-1))/(THETA(I
     1A(I-1))
      SA=(SIN((THETAS-PHIS)/RD))**2
      SB=(SIN(THETAS/RD))**2
      SC=VM2S**2
      UNULC=SQRT((2.+(GAM-1.)*SC*SA)/(SB*(2.*GAM*SC*SA-GAM+1.)))-V
      PRSHK=(2.*GAM*(VM1**2)*SB-GAM+1.)/(GAM+1.)
      TRSHK=PRSHK*((GAM-1.)*(VM1**2)*SB+2.)/((GAM+1.)*(VM1**2)*SB)
      DSR=(GAM/(GAM-1.))*ALOG(TRSHK)-ALOG(PRSHK)
      VM2C=VM2(N-1)+(VM2(N)-VM2(N-1))*(THE-THETA(N-1))/(THETA(N)-T
     1-1))
      PHIC=PHI(N-1)+(PHI(N)-PHI(N-1))*(THE-THETA(N-1))/(THETA(N)-T
     1-1))
      VMF=(1.+GF*SC)/(1.+GF*VM2C**2)
      PRC=PRSHK*(VMF**(GAM/(GAM-1.)))
      TRC=TRSHK*VMF
      VM2SRF=SQRT(USC**2/(GF*(1.-USC**2)))
      VMFS=(1.+GF*SC)/(1.+GF*VM2SRF**2)
      PRSRF=PRSHK*(VMFS**(GAM/(GAM-1.)))
      TRSRF=TRSHK*VMFS
2     RETURN
      END
```

AD-A085 327 AIR FORCE AERO PROPULSION LAB WRIGHT-PATTERSON AFB OH F/G 20/4
CALCULATION TECHNIQUES FOR INVISCID TWO-DIMENSIONAL SUPERSONIC --ETC(U)
SEP 79 M B BERGSTEN
UNCLASSIFIED AFAPL-TR-79-2023 NL

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TABLE A-8

PROGRAM FOR ILLUSTRATING THE USE OF SUBROUTINE CONSK

```
      PROGRAM CONES(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
    1 READ (5,*)VM1,DELT,GAM,THE
      CALL CONSK(VM1,DELT,GAM,THE,THETAS,DSR,VM2C,PHIC,PRC,TRC,VM2S,PRSH
     1K,TRSHK,PHIS,VM2SRF,PRSRF,TRSRF)
      WRITE (6,2)GAM,VM1,DELT,THETAS,DSR,THE,VM2C,PHIC,PRC,TRC,VM2SRF,PR
     1SRF,TRSRF,VM2S,PHIS,PRSHK,TRSHK
    2 FORMAT(1H1,54X,*CONE FLOW CALCULATION*,/,58X,*(GAMMA =*,F6.3,*)*,/
     1/,47X,*M1 =*,F6.3,12X,*DELTA =*,F6.2,*DEG*,//,56X,*THETA S =*,F7.3
     2,*DEG*,//,58X,*DS/R =*,F8.4,///,55X,*AT THETA =*,F7.3,*DEG:*,//,61
     3X,*M2 =*,F6.3,/,58X,*PHI =*,F7.3,*DEG*,/,58X,*P2/P1 =*,F7.4,/,58X,
     4*T2/T1 =*,F7.4,///,60X,*AT SURFACE:*,//,61X,*M2 =*,F6.3,/,58X,*P2/
     5P1 =*,F7.4,/,58X,*T2/T1 =*,F7.4,///,61X,*AT SHOCK:*,//,61X,*M2 =*,
     6F6.3,/,58X,*PHI =*,F7.3,*DEG*,/,58X,*P2/P1 =*,F7.4,/,58X,*T2/T1 =*
     7,F7.4)
      GO TO 1
      END
```

TABLE A-9

SUBROUTINE FOR THERMODYNAMIC PROPERTIES OF EQUILIBRIUM AIR

```
      SUBROUTINE THAIR (TC,PC,HC,SC,RHO,AM,CP,CV,GAM,A,K1,K2)
      T = TC
      P = PC
      H = HC
      S = SC
      IF (K2-1) 1,1,2
2     CP = .0685*GAM/(GAM-1.)
      IF (K1-1) 3,4,5
3     H = CP*T
      S = 1.63+CP*BLOG(T/500.)-.0685*BLOG(P)
      GO TO 6
4     T = H/CP
      S = 1.63+CP*BLOG(T/500.)-.0685*BLOG(P)
      GO TO 6
5     IF (K1-3) 7,8,9
7     T = 500.*EXX ((S-1.63+.0685*BLOG(P))/CP)
      H = CP*T
      GO TO 6
8     P = EXX ((1.63-S+CP*BLOG(T/500.))/.0685)
      H = CP*T
      GO TO 6
9     T = H/CP
      P = EXX ((1.63-S+CP*BLOG(T/500.))/.0685)
6     AM = 28.95
      CV = CP-.0685
      RHO = P*2116.*AM/(1545.*T)
      A = SQRT (32.174*GAM*1545.*T/AM)
      TC = T
      PC = P
      HC = H
      SC = S
      GO TO 42
1     IF (K1-1) 10,11,12
10    CALL EQNS (T,P,H,S,RHO,AM)
      HC = H
      SC = S
      GO TO 13
11    T1A = H/.24
      T2A = H/.3
      DO 14 I = 1,50
      CALL EQNS (T1A,P,H1A,S,RHO,AM)
      CALL EQNS (T2A,P,H2A,S,RHO,AM)
      IF (ABS ((H-H1A)/H)-.000001) 15,15,16
16    IF (I-50) 17,18,17
18    WRITE (6,19) T1A,T2A,H1A,H2A,H,P,K1
19    FORMAT (1H-,22HTHAIR DID NOT CONVERGE,1P6E16.4,5X,4HK1 =,I2)
      GO TO 42
```

TABLE A-9 (CONTINUED)

```
17    T1AN = T2A
      T2A = T2A+((T1A-T2A)*(H-H2A))/(H1A-H2A)
16    T1A = T1AN
15    T = T2A
      TO = T
      SO = S
      GO TO 13
12    IF (K1-3) 20,21,22
20    TA = 500.
      TB = 2000.
      DO 23 J = 1,50
      CALL EQNS (TA,P,H,SA,RHO,AM)
      CALL EQNS (TB,P,H,SB,RHO,AM)
      IF (ABS (S-SA)-.000001) 24,24,25
25    IF (J-50) 26,27,27
27    WRITE (6,19) S,SA,SB,TA,TB,K1
      GO TO 42
26    TAN = TB
      TB = TB+((TA-TB)*(S-SB))/(SA-SB)
23    TA = TAN
24    T = TB
      TO = T
      HO = H
      GO TO 13
21    PA = .001
      PB = 1.
      DO 28 M = 1,50
      CALL EQNS (T,PA,H,SA,RHO,AM)
      CALL EQNS (T,PB,H,SB,RHO,AM)
      IF (ABS (S-SA)-.000001) 29,29,30
30    IF (M-50) 31,32,32
32    WRITE (6,19) S,SA,SB,TA,PA,PB,K1
      GO TO 42
31    PAN = PB
      PB = EXX (BLOG(PB)+((BLOG(PA)-BLOG(PB))*(S-SB))/(SA-SB))
28    PA = PAN
29    P = PB
      PO = P
      HO = H
      GO TO 13
22    TA = H/.24
      DO 37 K = 1,50
      PA = .001
      PB = 1.
      DO 34 L = 1,50
      CALL EQNS (TA,PA,HA,SA,RHO,AM)
      CALL EQNS (TA,PB,HB,SB,RHO,AM)
      IF (ABS (S-SA)-.000001) 35,35,36
36    IF (L-50) 37,38,38
38    WRITE (6,19) S,SA,SB,TA,PA,PB,K1
      GO TO 42
37    PAN = PB
      PB = EXX (BLOG(PB)+((BLOG(PA)-BLOG(PB))*(S-SB))/(SA-SB))
34    PA = PAN
```

TABLE A-9 (CONTINUED)

```
35    P = PA
      CALL EQNS (TA,P,HC,S,RHO,AM)
      TC = TA*H/HC
      IF (ABS ((TA-TC)/TA)-.00001) 39,39,40
40    IF (K-50) 37,41,41
41    WRITE (6,19) TA,TC,H,P,HC,S,K1
      GO TO 42
37    TA = TA-(TA-TC)/2.
39    T = TA
      TT = T
      PP = P
13    IF (L2) 42,42,43
43    T1 = .999*T
      T2 = 1.001*T
      CALL EQNS (T1,P,H1,SX,RHOX,AMX)
      CALL EQNS (T2,P,H2,SX,RHOX,AMX)
      CP = (H2-H1)/(T2-T1)
      CV = CP-1.986/AM
      GAM = CP/CV
      A = SQRT (32.174*GAM*15+5.*T/AM)
42    RETURN
      END


      SUBROUTINE EQNS (T,P,H,S,RHO,AM)
      T = T/1.8
      V1 = 1.-EXX (-2275./T)
      V2 = 1.-EXY (-3390./T)
      Q1 = 3.+2.*EXX (-11390./T)+EXX (-18990./T)
      Q3 = 5.+3.*EXX (-228./T)+EXX (-326./T)+5.*EXX (-22800./T)
      Q4 = 4.+10.*EXX (-22700./T)+6.*EXX (-41500./T)
      Q5A = 4.+10.*EXX (-38600./T)+6.*EXX (-58200./T)
      Q5B = 1.+3.*EXX (-70.6/T)+5.*EXX (-188.9/T)+5.*EXX (-22600./T)+EXX
     1 (-47000./T)+5.*EXX (-67900./T)
      BQ1 = 3.5*BLOG(T)+.11-BLOG(V1)+BLOG(Q1)
      BQ2 = 3.5*BLOG(T)-.42-BLOG(V2)
      BQ3 = 2.5*BLOG(T)+.5+BLOG(Q3)
      BQ4 = 2.5*BLOG(T)+.3+BLOG(Q4)
      BQ5 = 2.5*BLOG(T)+.74+.2*BLOG(Q5A)+.8*BLOG(Q5B)
      BQ6 = 2.5*BLOG(T)-14.24
      AK = EXY (-59000./T+2.*BQ3-BQ1)
      BK = EXX (-113200./T+2.*BQ4-BQ2)
      GK = EXX (-166600./T+BQ5+BQ6-.2*BQ3-.8*BQ4)
      EA = (-.8+SZRT (.64+.8*(1.+4.*P/AK)))/(2.*(1.+4.*P/AK))
      IF (EA-.1999) 2,1,1
1     EA = 0
2     EB = (-.4+SZRT (.16+3.8+*(1.+4.*P/BK)))/(2.*(1.+4.*P/BK))
      IF (EB-.7999) 4,3,3
3     EB = 0.
4     EG = 1./SZRT (1.+P/GK)
      IF (EG-.9999) 6,5,5
5     EG = 0.
6     Z1= 1.+EA+EB+2.*EG
```

TABLE A-9 (CONTINUED)

```
      AM = 28.[illegible]/Z1
      R = 1545./AM
      V1P = (2270./T)*EXX (-2270./T)
      V2P = (3390./T)*EXX (-3390./T)
      Q1P = (22780./T)*EXX (-11390./T)+(18990./T)*EXX (-18990./T)
      Q3P = (684./T)*EXX (-224./T)+(326./T)*EXX (-326./T)+(114000./T)*EX
     1X (-22600./T)+(48600./T)*EXX (-48600./T)
      Q4P = (277000./T)*EXX (-27700./T)+(249000./T)*EXX (-41500./T)
      Q5AP = (336000./T)*EXX (-33600./T)+(349200./T)*EXX (-58200./T)
      Q5BP = (211.8/T)*EXX (-70.6/T)+(944.5/T)*EXX (-188.9/T)+(110000./T
     1)*EXX (-22000./T)+(470000./T)*EXX (-47000./T)+(339500./T)*EXX (-579
     200./T)
      H1 = 3.5+V1P/V1+Q1P/Q1
      H2 = 3.5+V2P/V2
      H3 = 2.5+Q3P/Q3
      H4 = 2.5+Q4P/Q4
      H5 = 2.5+.2*Q5AP/Q5A+.8*Q5BP/Q5B
      H6 = 2.5
      S1 = RQ1+H1
      S2 = RQ2+H2
      S3 = RQ3+H3
      S4 = RQ4+H4
      S5 = RQ5+H5
      S6 = RQ6+H6
      X1 = (.2-EA)/Z1
      X2 = (.8-EB)/Z1
      X3 = (2.*EA-.4*EG)/Z1
      X4 = (2.*EB-1.6*EG)/Z1
      X5 = 2.*EG/Z1
      X6 = X5
      IF (X1-1.E-6) 7,7,8
7     X1LG = 0
      GO TO 9
8     X1LG = BLOG(X1)
9     IF (X2-1.E-6) 10,10,11
10    X2LG = 0
      GO TO 12
11    X2LG = BLOG(X2)
12    IF (X3-1.E-6) 13,13,14
13    X3LG = 0
      GO TO 15
14    X3LG = BLOG(X3)
15    IF (X4-1.E-6) 16,16,17
16    X4LG = 0
      GO TO 18
17    X4LG = BLOG(X4)
18    IF (X5-1.E-6) 19,19,20
19    X5LG = 0
      GO TO 21
20    X5LG = BLOG(X5)
21    IF (X6-1.E-6) 22,22,23
22    X6LG = 0
      GO TO 24
```

TABLE A-9 (CONCLUDED)

```
23    X6LG = BLOG(X6)
24    S = R*(X1*(S1-X1LG)+X2*(S2-X2LG)+X3*(S3-X3LG)+X4*(S4-X4LG)+X5*(S5-
     1X5LG+.5004)+X6*(S6-X6LG)-BLOG(P))/778.
      TR = T*1.8
      H =R*TR*(X1*H1+X2*H2+X3*(H3+(29500./T))+X4*(H4+(56600./T))+X5*(H5+
     1218000./T))+X6*H6)/778.
      T = T*1.8
      RHO = P*2116./(R*T)
      RETURN
      END


      FUNCTION BLOG(XY)
      IF (XY-1.E-8) 1,2,2
1     XY= 1.E-8
2     BLOG = ALOG(XY)
      RETURN
      END


      FUNCTION SZRT(XY)
      IF (XY-1.E-8) 1,2,2
1     XY= 1.E-8
2     SZRT = SQRT(XY)
      RETURN
      END


      FUNCTION EXX(XY)
      SXY=SIGN(1.,XY)
      IF(ABS(XY)-88.) 2,1,1
1     XY=SXY*88.
2     EXX = EXP(XY)
      RETURN
      END
```

TABLE A-10

SUBROUTINE FOR NORMAL SHOCK CALCULATIONS

(REAL GAS)

```
      SUBROUTINE NRMSK (VM1,T1,P1A,VM2,T2,P2A,GAMI)
      IF (GAMI-.1) 1,2,2
1     K2 = 1
      GO TO 3
2     K2 = 2
3     CALL THAIR (T1,P1A,H1,S1,RH1,AM1,CP,CV,GAM,A1,0,K2)
      V1 = VM1*A1
      RH2A = RH1*6.*VM1**2/(VM1**2+5.)
      DO 4 I = 1,50
      P2A = (P1A*2116.+RH1*(V1**2)*(1.-RH1/RH2A)/32.174)/2116.
      H2 = H1+(V1**2)*(1.-(RH1/RH2A)**2)/50062.7
      CALL THAIR (T2,P2A,H2,S2,RHO2,AM2,CP,CV,GAM,A2,1,K2)
      IF (ABS((RHO2/RH1)-(RH2A/RH1))-.0001) 5,5,6
6     IF (I-20) 4,7,4
7     WRITE (6,8) RH2A,RH1,RHO2
8     FORMAT (1H0,10X,33HNRMSK DID NOT CONVERGE --- RH2A =,1PE11.4,3X,5H
     1RH1 =,E11.4,3X,6HRHO2 =,E11.4)
      RETURN
4     RH2A = RH2A+(RHO2-RH2A)/2.
5     VR = SQRT(1.-50062.7*(H2-H1)/(V1**2))
      VM2 = V1*VR/A2
      RETURN
      END
```

TABLE A-11
SUBROUTINE FOR OBLIQUE SHOCK CALCULATIONS
(REAL GAS)

```
      SUBROUTINE OSK(DELA,VMA,TA,PA,THET,VMB,TB,PB)
      EXTERNAL SKC
      COMMON DELT,RD,VM1,T1,P1,VM2,T2,P2,GAM1,RHO1,H1,THETA
      RD=.0174533
      DELT=DELA
      VM1=VMA
      T1=TA
      P1=PA
      THETA=THET
      CALL THAIR(T1,P1,H1,S1,RHO1,A1,CP,CV,GAM1,A1,0,1)
      CALL OGSK(DELT,VM1,T1,P1,GAM1,0,DELTM,THETA,VM2,T2,P2)
      IF(DELT-DELTM)1,1,2
2     TB=T1
      PB=P1
      VMB=VM1
      RETURN
1     URG=((VM2*SIN((THETA-DELT)*RD))/(VM1*SIN(THETA*RD)))*SQRT(T2/T1)
      URMX=1.+2.*((TAN(DELT*RD))**2)-2.*TAN(DELT*RD)*SQRT(1.+(TAN(DELT*
     1RD))**2)
      CALL NEWT1(UR,URG,.01,URMX,1.E-6,50,.001,SKC,SKC,K)
      VMB=VM2
      TB=T2
      PB=P2
      THET=THETA
      RETURN
      END
```

```
      FUNCTION SKC(UR,DUM1,DUM2,IT)
      COMMON DELT,RD,VM1,T1,P1,VM2,T2,P2,GAM1,RHO1,H1,THETA
      THETA=ATAN((1.-UR-SQRT((1.-UR)**2-4.*UR*((TAN(DELT*RD))**2)))/(2.*
     1UR*TAN(DELT*RD)))/RD
      P2=(1.+(1.-UR)*GAM1*(VM1**2)*(SIN(THETA*RD))**2)*P1
      H2=(1.+(1.-UR**2)*((GAM1-1.)/2.)*(VM1**2)*((SIN(THETA*RD))**2))*H1
      CALL THAIR(T2,P2,H2,S2,RHO2,A2,CP,CV,GAM2,A2,1,1)
      RHOR=RHO1/RHO2
      SKC=UR-RHOR
      RETURN
      END
```

TABLE A-12

SUBROUTINE FOR ISENTROPIC TURN CALCULATIONS

(REAL GAS)

```
      SUBROUTINE ISENXP(T1,P1,VM1,DEFL,DEFLN,GAM,VMU1,VMU2,T2,P2,VM2)
      RD=.0174533
      DEFLT=DEFL/DEFLN
      IF(GAM-.0001)1,1,2
1     K2=1
      GO TO 3
2     K2=2
3     VMU1=ASIN(1./VM1)/RD
      DEF=0.
      TA=T1
      PA=P1
      VMA=VM1
      GMA=GAM
      DO 4 J=1,1000
      DEF=DEF+DEFLT
      IF(DEF.LE.DEFL)GO TO 5
      CALL THATX(TA,PA,[illegible],[illegible],RHOA,[illegible],CP,CV,GMA,[illegible],[illegible],K2)
      CALL EXN(DEFLT,VMA,TA,PA,GMA,VMU,VMB,TB,PB)
      VMA=VMB
      TA=TB
4     PA=PB
5     VMU2=ASIN(1./VMB)/RD
      T2=TB
      P2=PB
      VM2=VMB
      RETURN
      END
```

TABLE A-13

PROGRAM FOR CONICAL FLOW CALCULATIONS

(REAL GAS)

```
      PROGRAM RCONE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION THETA(200),V2(200),U2(200),W2(200),PHI(200),VM2(200),RHO
     1W(200),RHO1W(200),T2(200),P2A(200),RHO2(200),H2(200),V1(200),P1AW(
     1200),T1W(200)
      GP=68080.134
      RD=57.29577951
      TGJ=50072.4
1     READ (5,*)DELT,DTHETA,USRF,TSRF,PSRFA
      IF(EOF(5)) 9,10
10    WRITE (6,2) DELT,USRF,TSRF,PSRFA
2     FORMAT(1H1,50X,*REAL GAS CONE FLOW CALCULATION*,///,51X,*CONE HALF
     1 ANGLE =*,F6.2,*DEGREES*,//,57X,*USURF =*,F7.1,*FPS*,//,55X,*TSURF
     2 =*,F7.1,*R*,7X,*PSURF =*,1PE10.3,*ATM*,///,25X,*THETA-DEG*,3X,*V2
     3-FPS*,4X,*U2-FPS*,4X,*W2-FPS*,3X,*PHI-DEG*,5X,*M2*,7X,*T2-R*,7X,*P
     42-ATM*,/)
      THETA(0)=DELT
      V2(0)=0.
      U2(0)=USRF
      P2A(0)=PSRFA
      T2(0)=TSRF
      CALL THAIR(T2(0),P2A(0),HSRF,S,RHO2(0),AM,CP,CV,GAM,A,0,0)
      HT=HSRF+(USRF**2)/TGJ
      DRHO=0.
      DO 3 I=1,200
      DERIV=-2.*U2(I-1)-V2(I-1)*((1./TAN(THETA(I-1)/RD))+DRHO/RHO2(I-1))
      THETA(I)=THETA(I-1)+DTHETA
      V2(I)=V2(I-1)+DTHETA*DERIV/RD
      U2(I)=U2(I-1)+DTHETA*V2(I-1)/RD+DERIV*((DTHETA/RD)**2)/2.
      W2(I)=SQRT((U2(I)**2)+(V2(I)**2))
      H2(I)=HT-(W2(I)**2)/TGJ
      CALL THAIR(T2(I),P2A(I),H2(I),S,RHO2(I),AM,CP,CV,GAM,A2,4,1)
      VM2(I)=W2(I)/A2
      PHI(I)=ATAN(V2(I)/U2(I))*RD+THETA(I)
      DRHO=(RHO2(I)-RHO2(I-1))*RD/DTHETA
      UW2=W2(I)*SIN((THETA(I)-PHI(I))/RD)
      U1=UW2*TAN(THETA(I)/RD)/TAN((THETA(I)-PHI(I))/RD)
      V1(I)=U1/SIN(THETA(I)/RD)
      RHO1W(I)=RHO2(I)*UW2/U1
      P1AW(I)=P2A(I)+(RHO2(I)*(UW2**2)-RHO1W(I)*(U1**2))/GP
      H1W=HT-(V1(I)**2)/TGJ
      CALL THAIR(T1W(I),P1AW(I),H1W,SW,RHOW(I),AM,CP,CV,GAM,A,1,0)
      IF(RHO1W(I)-RHOW(I))5,4,4
5     WRITE (6,6)THETA(I),V2(I),U2(I),W2(I),PHI(I),VM2(I) ,T2(I),P2A(I)
6     FORMAT(1H ,F31.2,F11.1,2F10.1,F9.2,F10.4,F10.1,1PE14.4)
3     CONTINUE
4     THETAS=THETA(I-1)+DTHETA*(RHOW(I-1)-RHO1W(I-1))/((RHOW(I-1)-RHO1W(
     1I-1))-(RHOW(I)-RHO1W(I)))
      THEF=(THETAS-THETA(I-1))/(THETA(I)-THETA(I-1))
```

TABLE A-13 (CONCLUDED)

```
      PHIS=PHI(I-1)+(PHI(I)-PHI(I-1))*THEF
      T2S=T2(I-1)+(T2(I)-T2(I-1))*THEF
      P2SA=P2A(I-1)+(P2A(I)-P2A(I-1))*THEF
      T1=T1W(I-1)+(T1W(I)-T1W(I-1))*THEF
      P1A=P1AW(I-1)+(P1AW(I)-P1AW(I-1))*THEF
      CALL THAIR(T1,P1A,H1,S1,RHO1,AM,CP,CV,GAM,A1,0,1)
      VM1=V1(I)/A1
      WRITE (6,7)VM1,THETAS
 7    FORMAT(1H0,49X,*M1 =*,F7.4,6X,*THETA S =*,F7.3,*DEG*)
      PRSHK=P2SA/P1A
      TRSHK=T2S/T1
      PRSRF=PSRFA/P1A
      TRSRF=TSRF/T1
      DSR=(S-S1)/.0685
      WRITE (6,8)PRSHK,PRSRF,TRSHK,TRSRF,DSR
 8    FORMAT(1H0,41X,*(P2/P1)SHK =*,F7.3,13X,*(P2/P1)SURF =*,F7.3,/,
     142X,*(T2/T1)SHK =*,F7.3,13X,*(T2/T1)SURF =*,F7.3,//,61X,*DS/R =*,F
     28.4)
      GO TO 1
 9    STOP
      END
```

TABLE A-14

SUBROUTINE FOR CONICAL SHOCK CALCULATIONS

(REAL GAS)

```
      SUBROUTINE RSCON(VM1S,T1S,P1AS,DELTS,THES,THETASS,CSKS,VM2CS,PHICS
     1,PRCS,TRCS,VM2SS,PRSHKS,TRSHKS,PHISS,VM2SRFS,PRSRFS,TRSRFS)
      EXTERNAL RONC3
      COMMON D(11),THE,DALT,VM1,TA,P1,THETAS,CSK,VM2C,PHIC,PRC,TRC,VM2S,
     1TRSHK,PHIS,PRSRF,TRSRF,PRSHK,TRSF,PRSF,D1(3)
      VM1=VM1S
      TA=T1S
      P1=P1AS
      DALT=DELTS
      THE=THES
      VMS=VM1*(1.-(DALT/200.)-((DALT**2)/5000.))
      SF=DALT*(VM1**2)/SQRT((VM1**2)-1.)
      PRSRF=1.+.024*SF
      DO 1 I=1,50
      TRSRF=1.+.007*SF
      DO 2 J=1,50
      USRFG=48.*VMS*SQRT(TRSRF*TA)
      USRFMN=100.
      USRFMX=50.*VM1*SQRT(TRSRF*TA*10.)
      CALL NEWTN(USRF,USRFG,USRFMN,USRFMX,1.E-5,50,10.,RONC3,
      RONC2=TRSF-TRSRF
      IF(J-50) 6,5,6
5     WRITE (6,7)
7     FORMAT(1H ,*TEMP ITERATION DID NOT CONVERGE*)
6     IF(ABS(RONC2)-1.E-5) 3,3,2
2     TRSRF=TRSF
3     RONC1=PRSF-PRSRF
      SF=(TRSF-1.)/.007
      IF(I-50) 8,9,8
9     WRITE (6,10)
10    FORMAT (1H ,*PRESSURE ITERATION DID NOT CONVERGE*)
8     IF(ABS(RONC1)-1.E-5) 4,4,1
1     PRSRF=PRSF
4     THETASS=THETAS
      CSKS=CSK
      VM2CS=VM2C
      PHICS=PHIC
      PRCS=PRC
      TRCS=TRC
      VM2SS=VM2S
      PRSHKS=PRSHK
      TRSHKS=TRSHK
      PHISS=PHIS
      VM2SRFS=VM2SRF
      PRSRFS=PRSRF
      TRSRFS=TRSRF
      RETURN
      END
```

TABLE A-14 (CONTINUED)

```
      FUNCTION [illegible](USRF,UUM4,DJ12,N[illegible])
      COMMON D(11),THE,DALT,VM1,T[illegible],P1,THETAS, [illegible]
     1T[illegible]HK,[illegible],[illegible]RF,TDSRF,[illegible],[illegible]
      DIMENSION THETA(200),V2(200),U2(200),W2(200),PHI(200),VM2(200),T1W
     1(200),T2(200),P2A(200),RHO2(200),H2(200),V1(200),P1AW(200),RHOW(
     120),RHO1W(200),A(200),SW(200)
      SF=[illegible].1[illegible]
      [illegible]=H7.29[illegible]
      TGJ=[illegible].4
      DTHETA=.2
      THETA(0)=[illegible]
      V2(0)=0.
      U2(0)=[illegible]
      P2A(0)=P[illegible]*P1
      T2(0)=T[illegible]*TA
      CALL THAIR(T2(0),P2A(0),H[illegible],S,RHO2(0),AM,CP,CV,GAM,A(0),0,1)
      HT=HS[illegible]+(US[illegible]**2)/TGJ
      DRHO=0.
      N=0
      DO 3 I=1,200
      DERIV=-2.*U2(I-1)-V2(I-1)*((1./TAN(THETA(I-1)/RD))+DRHO/RHO2(I-1))
      THETA(I)=THETA(I-1)+DTHETA
      IF(N)1,5,1
5     IF(THE-THETA(I)) 2,1,1
2     N=I
1     V2(I)=V2(I-1)+DTHETA*DERIV/RD
      U2(I)=U2(I-1)+DTHETA*V2(I-1)/RD+DERIV*((DTHETA/RD)**2)/2.
      W2(I)=SQRT((U2(I)**2)+(V2(I)**2))
      H2(I)=HT-(W2(I)**2)/TGJ
      IF(H2(I))7,7,8
7     H2(I)=1.
8     CALL THAIR(T2(I),P2A(I),H2(I),S,RHO2(I),AM,CP,CV,GAM,A2,0,1)
      VM2(I)=W2(I)/A2
      PHI(I)=ATAN(V2(I)/U2(I))*RD+THETA(I)
      DRHO=(RHO2(I)-RHO2(I-1))*RD/DTHETA
      UW2=W2(I)*SIN((THETA(I)-PHI(I))/RD)
      U1=UW2*TAN(THETA(I)/RD)/TAN((THETA(I)-PHI(I))/RD)
      V1(I)=U1/SIN(THETA(I)/RD)
      RHO1W(I)=RHO2(I)*UW2/U1
      P1AW(I)=P2A(I)+(RHO2(I)*(UW2**2)-RHO1W(I)*(U1**2))/GF
      H1W=HT-(V1(I)**2)/TGJ
      IF(H1W)9,9,6
9     H1W=1.
6     CALL THAIR(T1W(I),P1AW(I),H1W,SW(I),RHOW(I),AM,CP,CV,GAM,A(I),1,1)
      IF(RHO1W(I)-RHOW(I))3,4,4
3     CONTINUE
4     THETAS=THETA(I-1)+DTHETA*(RHOW(I-1)-RHO1W(I-1))/((RHOW(I-1)-RHO1W(
     1I-1))-(RHOW(I)-RHO1W(I)))
      THEF=(THETAS-THETA(I-1))/DTHETA
      PHIS=PHI(I-1)+(PHI(I)-PHI(I-1))*THEF
      T2S=T2(I-1)+(T2(I)-T2(I-1))*THEF
      P2SA=P2A(I-1)+(P2A(I)-P2A(I-1))*THEF
      VM1C=V1(I-1)/A(I-1)+(V1(I)/A(I)-V1(I-1)/A(I-1))*THEF
      RONC3=VM1-VM1C
      TAC=T1W(I-1)+(T1W(I)-T1W(I-1))*THEF
      P1C=P1AW(I-1)+(P1AW(I)-P1AW(I-1))*THEF
      TDSF=T2(0)/TAC
```

TABLE A-14 (CONCLUDED)

```
PRSF=P2A(I)/P10
VM2SRF=USRF/A(I)
PRSHK=P2SA/P1
TRSHK=T2S/TA
VM2S=VM2(I-1)+(VM2(I)-VM2(I-1))*THEF
SWI=SW(I-1)+(SW(I)-SW(I-1))*THEF
DSN=(S-SWI)/.0685
THEN=(THE-THETA(N-1))/(THETA(N)-THETA(N-1))
VM2C=VM2(N-1)+(VM2(N)-VM2(N-1))*THEN
PHIC=PHI(N-1)+(PHI(N)-PHI(N-1))*THEN
T2C=T2(N-1)+(T2(N)-T2(N-1))*THEN
P2CA=P2A(N-1)+(P2A(N)-P2A(N-1))*THEN
TRC=T2C/TA
PRC=P2CA/P1
RETURN
END
```

TABLE A-15

PROGRAM FOR ILLUSTRATING THE USE OF SUBROUTINE RGCON

```
      PROGRAM RGCONE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
1     READ (5,*)VM1,TA,P1A,DALT,THE
      IF(EOF(5)) 3,4
4     CALL RGCON(VM1,TA,P1A,DALT,THE,THETAS,DSR,VM2C,PHIC,PRC,TRC,VM2S,P
     1RSHK,TRSHK,PHIS,VM2SRF,PRSRF,TRSRF)
      WRITE (6,2)VM1,TA,P1A,DALT,THETAS,DSR,THE,VM2C,PHIC,PRC,TRC,VM2SRF
     1,PRSRF,TRSRF,VM2S,PHIS,PRSHK,TRSHK
2     FORMAT(1H1,50X,*REAL GAS CONE FLOW CALCULATION*,//,51X,*M1 =*,F6.
     1,/,48X,*T1 =*,F6.1,* K*,7X,*P1 =*,1PE11.4,*ATM*,/,53X,*DELTA =*,0P
     2F6.2,*DEG*,//,50X,*THETA S =*,F7.3,*DEG*,//,53X,*DS/R =*,F6.4,///,
     35X,*AT THETA =*,F7.3,*DEG:*,//,61X,*M2 =*,F6.3,/,55X,*PHI =*,F7.3,
     4*DEG*,/,58X,*P2/P1 =*,F7.4,/,58X,*T2/T1 =*,F7.4,///,60X,*AT SURFAC
     5E:*,//,61X,*M2 =*,F6.3,/,58X,*P2/P1 =*,F7.4,/,58X,*T2/T1 =*,F7.4,/
     6//,61X,*AT SHOCK:*,//,61X,*M2 =*,F6.3,/,55X,*PHI =*,F7.3,*DEG*,/,
     73X,*P2/P1 =*,F7.4,/,58X,*T2/T1 =*,F7.4)
      GO TO 1
3     STOP
      END
```

TABLE A-16

IDEAL GAS PROGRAM FOR PROBLEM 1 - SWEPT WING

```
      PROGRAM WING(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL M0,LAMBDA,ME0,M1,M2,M3,M4,LIFT
      RD=.01745333
      PC=2116.
1     READ(5,2)M0,T0,P0,Y1,Y2,SIG1,SIG2,SIG3,SIG4,LAMBDA,ALPHA
2     FORMAT(12F6.0)
      IF(EOF(5))4,3
3     CALL SWEP(LAMBDA,M0,ALPHA,SIG1,ME0,ALPE,SIGE1)
      CALL SWEP(LAMBDA,M0,ALPHA,SIG2,ME0,ALPE,SIGE2)
      CALL SWEP(LAMBDA,M0,ALPHA,SIG3,ME0,ALPE,SIGE3)
      CALL SWEP(LAMBDA,M0,ALPHA,SIG4,ME0,ALPE,SIGE4)
      DELTE1=SIGE1-ALPE
      DELTE2=SIGE2+ALPE
      DELTE3=-SIGE1-SIGE3
      DELTE4=-SIGE2-SIGE4
      X1=Y1/TAN(SIG1*RD)
      X2=Y2/TAN(SIG2*RD)
      X3=(Y1+Y2+(X2-X1)*TAN(SIG4*RD))/(TAN(SIG3*RD)+TAN(SIG4*RD))
      X4=X1+X3-X2
      X=X1+X3
      Y3=X3*TAN(SIG3*RD)
      Y4=X4*TAN(SIG4*RD)
      CALL OGSK(DELTE1,ME0,T0,P0,1.4,0,DELTM,THE1,M1,T1,P1)
      CALL OGSK(DELTE2,ME0,T0,P0,1.4,0,DELTM,THE2,M2,T2,P2)
      CALL EXN(DELTE3,M1,T1,P1,1.4,VMU3,M3,T3,P3)
      CALL EXN(DELTE4,M2,T2,P2,1.4,VMU4,M4,T4,P4)
      F=PC*(P3*Y3+P4*Y4-P1*Y1-P2*Y2)
      N=PC*(P2*X2+P4*X4-P1*X1-P3*X3)
      DRAG=N*SIN(ALPHA*RD)-F*COS(ALPHA*RD)
      LIFT=N*COS(ALPHA*RD)+F*SIN(ALPHA*RD)
      WRITE(6,5)M0,P0,T0,ALPHA,Y1,Y2,SIG1,SIG2,SIG3,SIG4,LAMBDA,ME0,SIGE
     11,SIGE2,SIGE3,SIGE4,DELTE1,DELTE2,DELTE3,DELTE4,X1,X2,X3,X4,X,Y3,Y
     24,DELTM,M1,M2,M3,M4,T1,T2,T3,T4,P1,P2,P3,P4,DRAG,LIFT
5     FORMAT(1H1,19X,*PRESSURE DISTRIBUTION OVER SWEPT WING*/30X,*OF INF
     1INITE SPAN*,///,19X,*M0=*,F4.1,3X,*P0=*,1PE8.2,*ATM*,3X,*T0=*,0PF5
     2.0,*R*,//31X,*ALPHA=*,F5.1,*DEG*,//27X,*Y1=*,F4.2,*FT*,3X,*Y2=*,F4
     3.2,*FT*//11X,*SIG1=*,F4.1,*DEG*,3X,*SIG2=*,F4.1,*DEG*,3X,*SIG3=*,F
     44.1*DEG*3X*SIG4=*,F4.1,*DEG*,//30X,*LAMBDA=*,F5.1,*DEG*,///,33X,*M
     5E0=*,F6.3,//7X,*SIGE1=*,F5.2,*DEG*,3X,*SIGE2=*,F5.2,*DEG*,2X,*SIGE
     63=*,F5.2,*DEG*,3X,*SIGE4=*,F5.2,*DEG*,//5X,*DELTE1=*,F5.2,*DEG*,2X
     7,*DELTE2=*,F5.2,*DEG*,2X,*DELTE3=*,F6.2,*DEG*,2X,*DELTE4=*,F6.2,*D
     8EG*//9X,*X1=*,F5.2,*FT*,7X,*X2=*,F5.2,*FT*,6X,*X3=*,F5.2,*FT*,7X,*
     9X4=*,F5.2,*FT*,//29X,*TOTAL LENGTH=*,F5.2,*FT*,//27X,*Y3=*,F4.2,*F
     AT*,5X,*Y4=*,F4.2,*FT*,///30X,*DELTMAX=*,F5.2,*DEG*,//9X,*M1=*,F5.2
     1, 9X,*M2=*,F5.2,8X,*M3=*,F5.2, 9X,*M4=*F5.2,//8X,*T1=*,F6.1,*R*, 7
     2X,*T2=*,F6.1,*R*,6X,*T3=*,F6.1,*R*, 8X,*T4=*,F6.1,*R*//5X,*P1=*,1P
     3E9.3,*ATM*,2X,*P2=*,E9.3,*ATM*,2X,*P3=*,E9.3,*ATM*,2X,*P4=*.E9.3,*
     4ATM*,///,22X,*DRAG=*,0PF8.1,*LBS/FT OF WING SPAN*,//22X,*LIFT=*,FP
     55.1,*LBS/FT OF WING SPAN*)
      GO TO 1
4     STOP
      END
```

TABLE A-17

REAL GAS PROGRAM FOR PROBLEM 1 - SWEPT WING

```
      PROGRAM WING(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL M0,LAMBDA,ME0,M1,M2,M3,M4,LIFT
      RD=.0174533
      PC=2116.
1     READ(5,2)M0,T0,P0,Y1,Y2,SIG1,SIG2,SIG3,SIG4,LAMBDA,ALPHA
2     FORMAT(12E6.0)
      IF(EOF(5))4,3
3     CALL SWEP(LAMBDA,M0,ALPHA,SIG1,ME0,ALPE,SIGE1)
      CALL SWEP(LAMBDA,M0,ALPHA,SIG2,ME0,ALPE,SIGE2)
      CALL SWEP(LAMBDA,M0,ALPHA,SIG3,ME0,ALPE,SIGE3)
      CALL SWEP(LAMBDA,M0,ALPHA,SIG4,ME0,ALPE,SIGE4)
      DELTE1=SIGE1-ALPE
      DELTE2=SIGE2+ALPE
      DELTE3=-SIGE1-SIGE3
      DELTE4=-SIGE2-SIGE4
      X1=Y1/TAN(SIG1*RD)
      X2=Y2/TAN(SIG2*RD)
      X3=(Y1+Y2+(X2-X1)*TAN(SIG4*RD))/(TAN(SIG3*RD)+TAN(SIG4*RD))
      X4=X1+X3-X2
      X=X1+X3
      Y3=X3*TAN(SIG3*RD)
      Y4=X4*TAN(SIG4*RD)
      CALL RGSK(DELTE1,ME0,T0,P0,THE1,M1,T1,P1)
      CALL RGSK(DELTE2,ME0,T0,P0,THE2,M2,T2,P2)
      CALL RGEXP(T1,P1,M1,DELTE3,20,0,VMU1,VMU3,T3,P3,M3)
      CALL RGEXP(T2,P2,M2,DELTE4,20,0,VMU2,VMU4,T4,P4,M4)
      F=PC*(P3*Y3+P4*Y4-P1*Y1-P2*Y2)
      N=PC*(P2*X2+P4*X4-P1*X1-P3*X3)
      DRAG=N*SIN(ALPHA*RD)-F*COS(ALPHA*RD)
      LIFT=N*COS(ALPHA*RD)+F*SIN(ALPHA*RD)
      WRITE(6,5)M0,P0,T0,ALPHA,Y1,Y2,SIG1,SIG2,SIG3,SIG4,LAMBDA,ME0,SIGE
     11,SIGE2,SIGE3,SIGE4,DELTE1,DELTE2,DELTE3,DELTE4,X1,X2,X3,X4,X,Y3,Y
     24,DELTM,M1,M2,M3,M4,T1,T2,T3,T4,P1,P2,P3,P4,DRAG,LIFT
5     FORMAT(1H1,19X,*PRESSURE DISTRIBUTION OVER SWEPT WING*/30X,*OF INF
     1INITE SPAN*,///,13X,*M0=*,F4.1,3X,*P0=*,1PE8.2,*ATM*,3X,*T0=*,0PF5
     2.0,*R*,//31X,*ALPHA=*,F5.1,*DEG*,//27X,*Y1=*,F4.2,*FT*,3X,*Y2=*,F4
     3.2,*FT*//11X,*SIG1=*,F4.1,*DEG*,3X,*SIG2=*,F4.1,*DEG*,3X,*SIG3=*,F
     44.1*DEG*3X*SIG4=*,F4.1,*DEG*,//30X,*LAMBDA=*,F5.1,*DEG*,///,33X,*M
     5E0=*,F6.3,//7X,*SIGE1=*,F5.2,*DEG*,3X,*SIGE2=*,F5.2,*DEG*,2X,*SIGE
     63=*,F5.2,*DEG*,3X,*SIGE4=*,F5.2,*DEG*,//5X,*DELTE1=*,F5.2,*DEG*,2X
     7,*DELTE2=*,F5.2,*DEG*,2X,*DELTE3=*,F6.2,*DEG*,2X,*DELTE4=*,F6.2,*D
     8EG*//9X,*X1=*,F5.2,*FT*,7X,*X2=*,F5.2,*FT*,6X,*X3=*,F5.2,*FT*,7X,*
     9X4=*,F5.2,*FT*,//29X,*TOTAL LENGTH=*,F5.2,*FT*,//27X,*Y3=*,F4.2,*F
     AT*,5X,*Y4=*,F4.2,*FT*,///30X,*DELTMAX=*,F5.2,*DEG*,//9X,*M1=*,F5.2
     1, 9X,*M2=*,F5.2,8X,*M3=*,F5.2, 9X,*M4=*F5.2,//8X,*T1=*,F6.1,*R*, 7
     2X,*T2=*,F6.1,*R*,6X,*T3=*,F6.1,*R*, 6X,*T4=*,F6.1,*R*//5X,*P1=*,1P
     3E9.3,*ATM*,2X,*P2=*,E9.3,*ATM*,2X,*P3=*,E9.3,*ATM*,2X,*P4=*,E9.3,*
     4ATM*,///,22X,*DRAG=*,0PF8.1,*LBS/FT OF WING SPAN*,//22X,*LIFT=*,F8
     5.1,*LBS/FT OF WING SPAN*)
      GO TO 1
4     STOP
      END
```

TABLE A-18

PROGRAM FOR SOLUTION OF PROBLEM 2- 2-D INLET

TABLE A-19

PROGRAM FOR SOLUTION OF PROBLEM 3 - CONICAL INLET

```
      PROGRAM INL3D(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL M0,M1,M2
    1 READ(5,2)M0,T0,P0,DELT,CWLANG
    2 FORMAT(5E10.0)
      IF(EOF(5))4,3
    3 THE=DELT+(CWLANG-DELT)/2.
      CALL CONSK(M0,DELT,1.4,THE,THETAS,DSR,M1,PHI,PR1,TR1,VMIS,PRSHK,TR
     1SHK,PHIS,VM2SRF,PRSRF,TRSRF)
      T1=TR1*T0
      P1=PR1*P0
      CALL NORM(M1,T1,P1,M2,T2,P2,1.4)
      ETAR=(P2*((1.+.2*(M2**2))**3.5))/(P0*((1.+.2*(M0**2))**3.5))
      WRITE(6,5)M0,P0,T0,DELT,CWLANG
    5 FORMAT(1H1,17X,*BACK PRESSURE AND PRESSURE RECOVERY*,//,27X,*FOR C
     1ONICAL INLET*,///,15X,*M0=*,F4.1,4X,*P0=*,1PE8.2,*ATM*,4X,*T0=*,0P
     2F6.1,*R*,//,28X,*DELTA=*F4.1,*DEG*,///,26X,*COWL ANGLE=*,F5.2,*DEG
     3*,////,26X,*IDEAL GAS SOLUTION*,/)
      WRITE(6,6)M1,P1,T1
      WRITE(6,7)M2,P2,T2
    6 FORMAT(1H ,13X,*M1=*,F5.2,4X,*P1=*,1PE9.3,*ATM*,4X,*T1=*0PF7.1*R*)
    7 FORMAT(1H ,13X,*M2=*,F5.2,4X,*P2=*,1PE9.3,*ATM*,4X,*T2=*0PF7.1*R*)
      WRITE(6,8)THETAS,P2,ETAR
    8 FORMAT(1H ,/,28X,*THETAS=*,F5.2,*DEG*,//,17X,*REQUIRED BACK PRESSU
     1RE=*,1PE9.3,*ATM*,//,19X,*PRESSURE RECOVERY(PT2/PT0)=*,0PF5.4////)
      CALL THAIR(T0,P0,H0,S0,RH0,AM0,CP,CV,GAM,A0,0,1)
      V0=M0*A0
      HT=H0+(V0**2)/50072.
      CALL THAIR(TT0,PT0,HT,S0,RHT,AMT,CP,CV,GAM,AT0,4,0)
      CALL RGCON(M0,T0,P0,DELT,THE,THETAS,DSR,M2,PHI,PR1,TR1,VM2S,PRSHK,
     1TRSHK,PHIS,VM2SRF,PRSRF,TRSRF)
      CALL NRMLK(M1,T1,P1,M2,T2,P2,0)
      CALL THAIR(T2,P2,H2,S2,RH2,AM2,CP,CV,GAM,A2,0,0)
      WRITE(6,9)
    9 FORMAT(1H ,26X,*REAL GAS SOLUTION*,/)
      WRITE(6,6)M1,P1,T1
      WRITE(6,7)M2,P2,T2
      CALL THAIR(TT2,PT2,HT,S2,RHT,AMT,CP,CV,GAM,AT3,4,0)
      ETAR=PT2/PT0
      WRITE(6,8)THETAS,P2,ETAR
      GO TO 1
    4 STOP
      END
```

APPENDIX B

CHARTS FOR CALCULATION OF LOW MACH NUMBER SUPERSONIC AIRFLOWS

In this appendix charts are included for calculations involving normal shock waves, two-dimensional oblique shock waves, swept wings, isentropic expansions, and supersonic flow over zero angle of attack cones. The data for the charts has been calculated by use of the programs and subroutines discussed in Section I of the text. The charts are to be used only in cases for which the ideal gas relations are valid.

Copies of these working charts plotted on graph paper are available directly from the author at the following address:

AFAPL/RJT
ATTN: M. B. Bergsten
WPAFB OH 45433

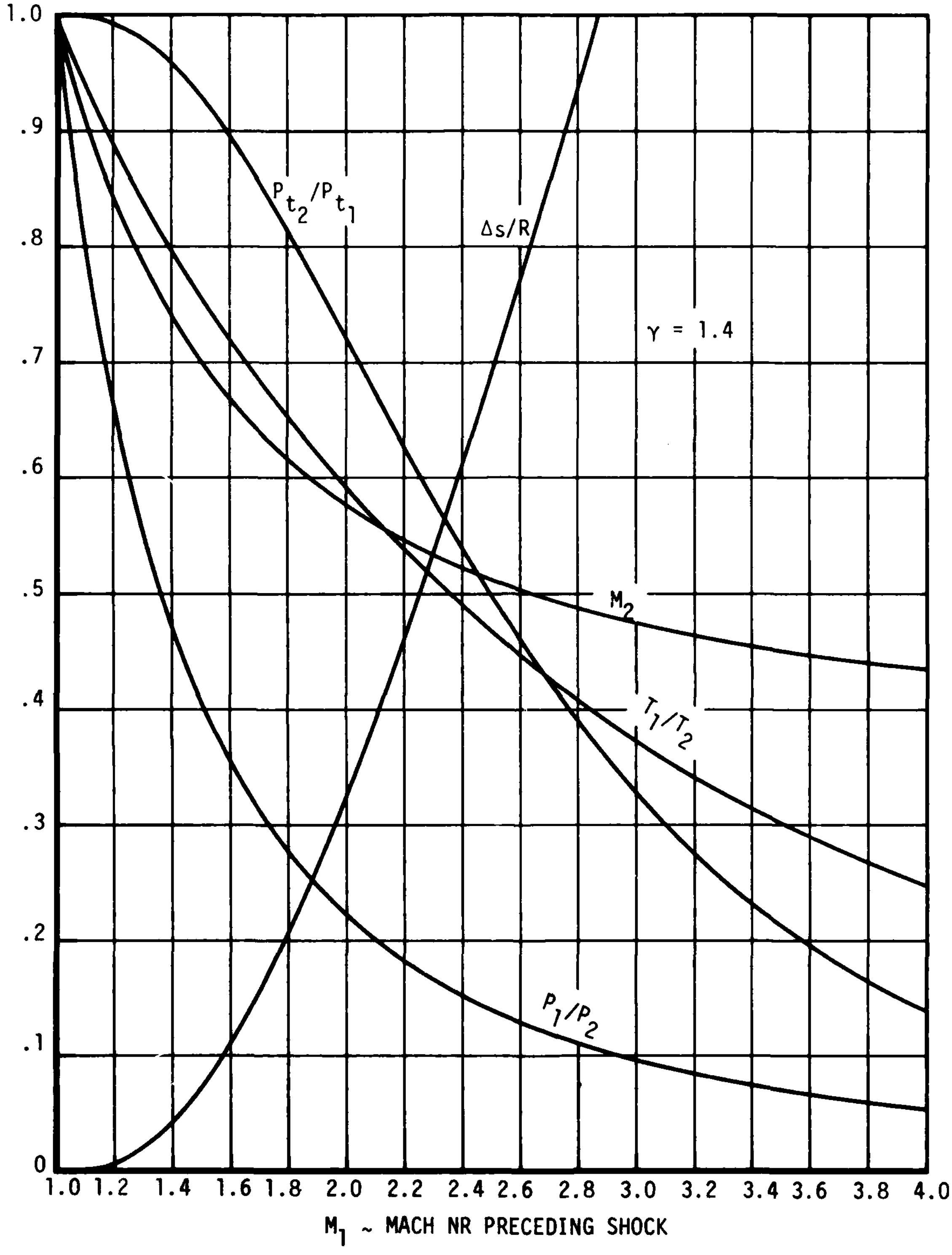


Figure B-1. Parameters for Normal Shock Calculations

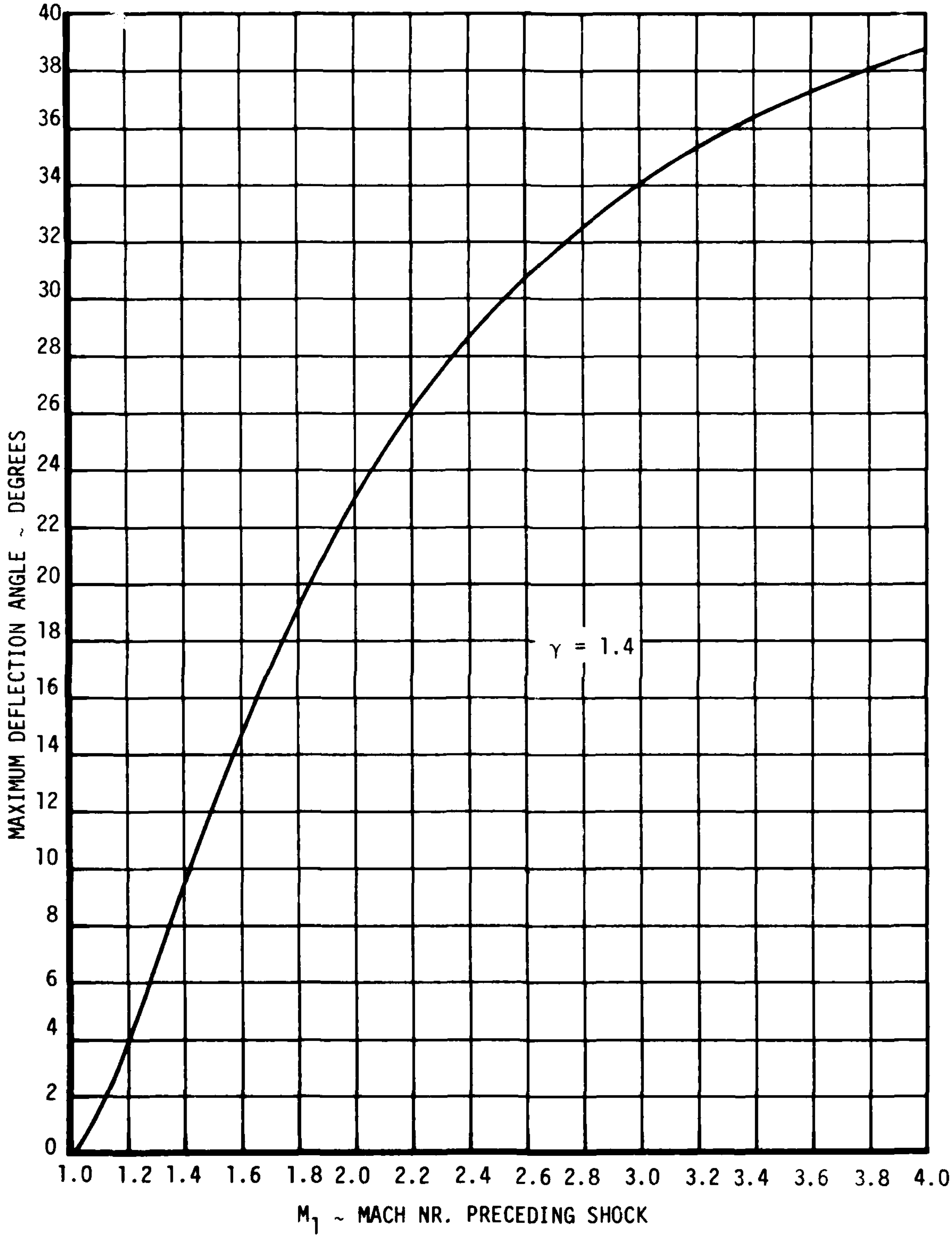


Figure B-2. Maximum Deflection Angle for 2-D Wedges

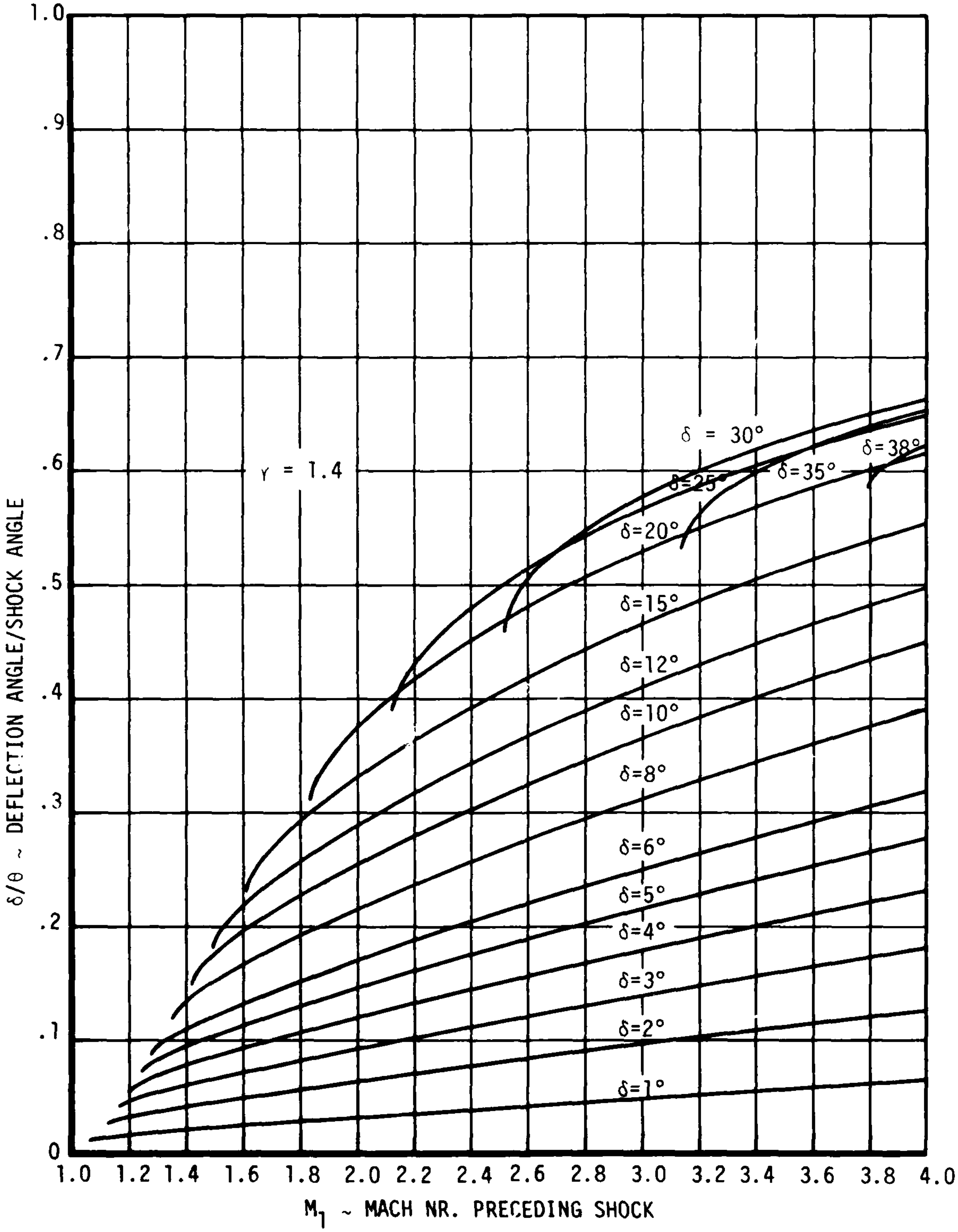


Figure B-3. Shock Wave Angle for 2-D Deflections

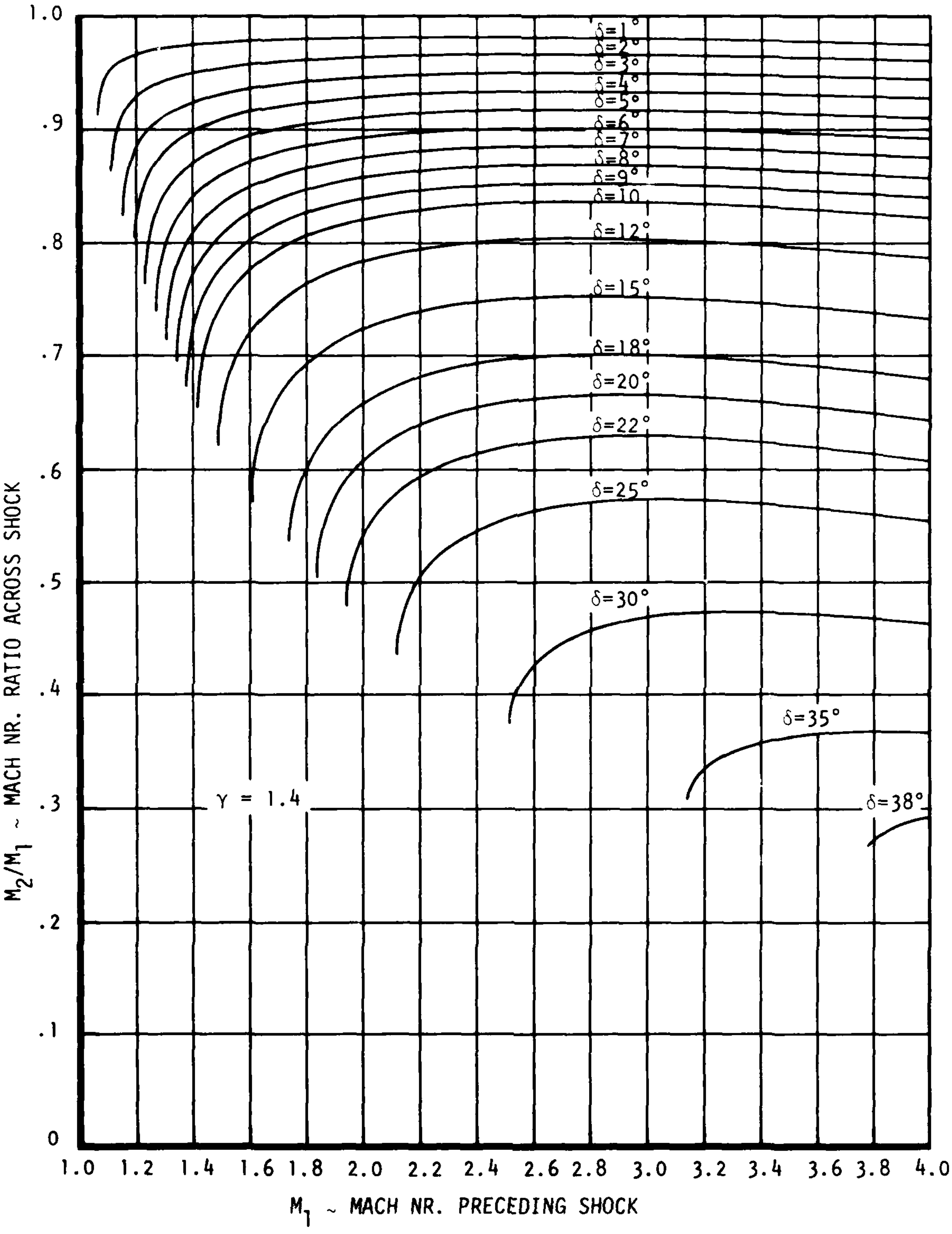


Figure B-4. Mach Number Ratio for 2-D Deflections

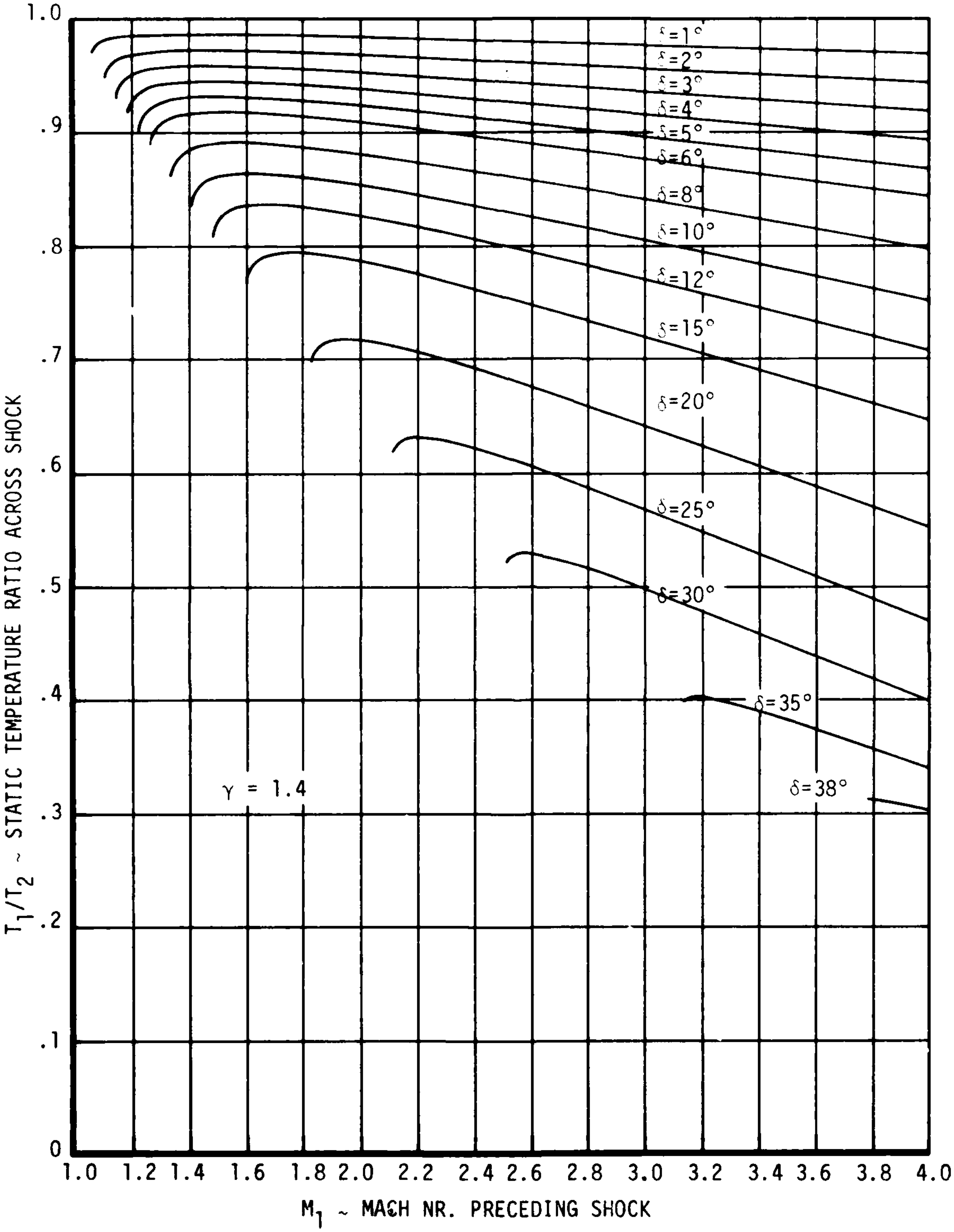


Figure B-5. Static Temperature Ratio for 2-D Deflections

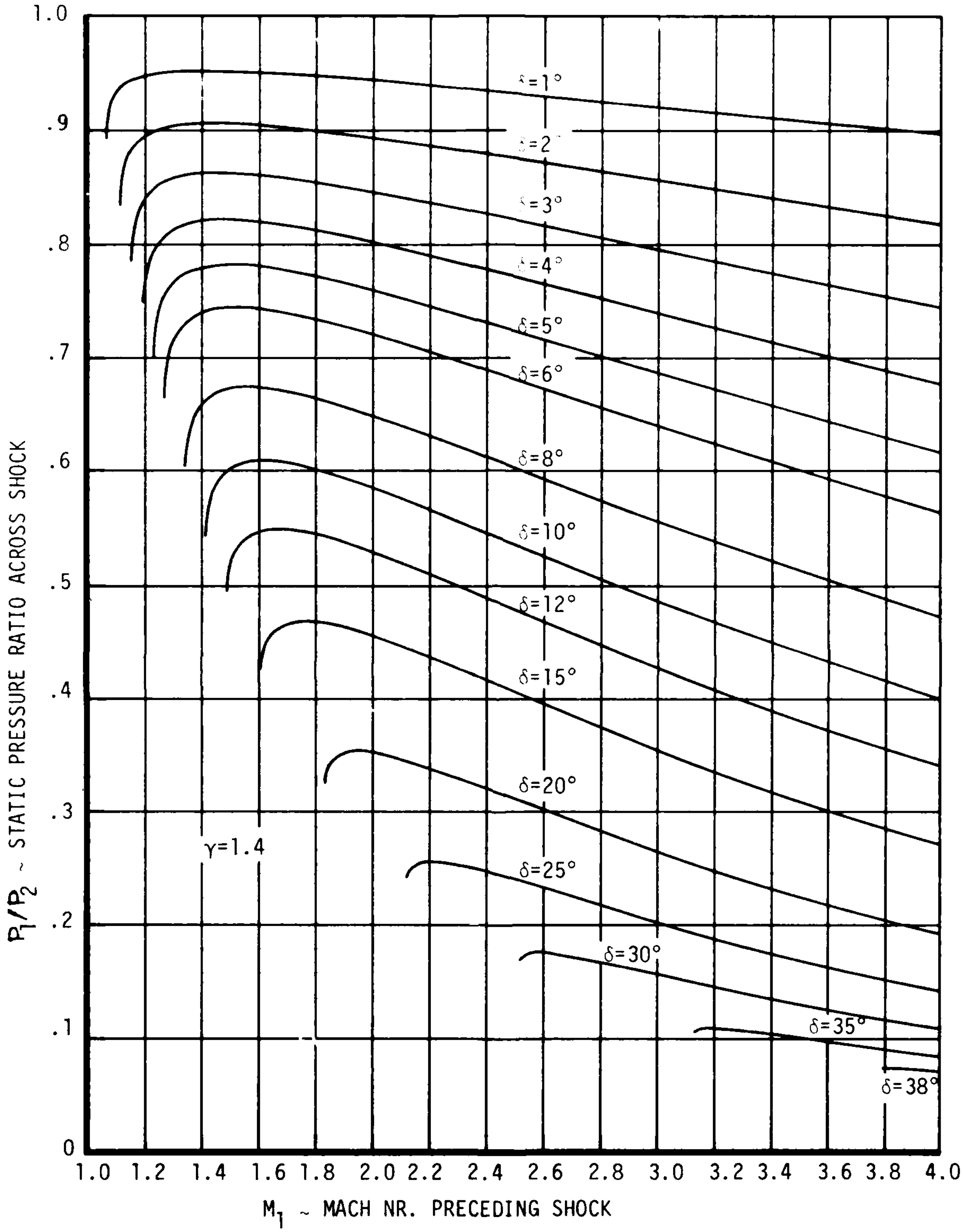


Figure B-6. Static Pressure Ratio for 2-D Deflections

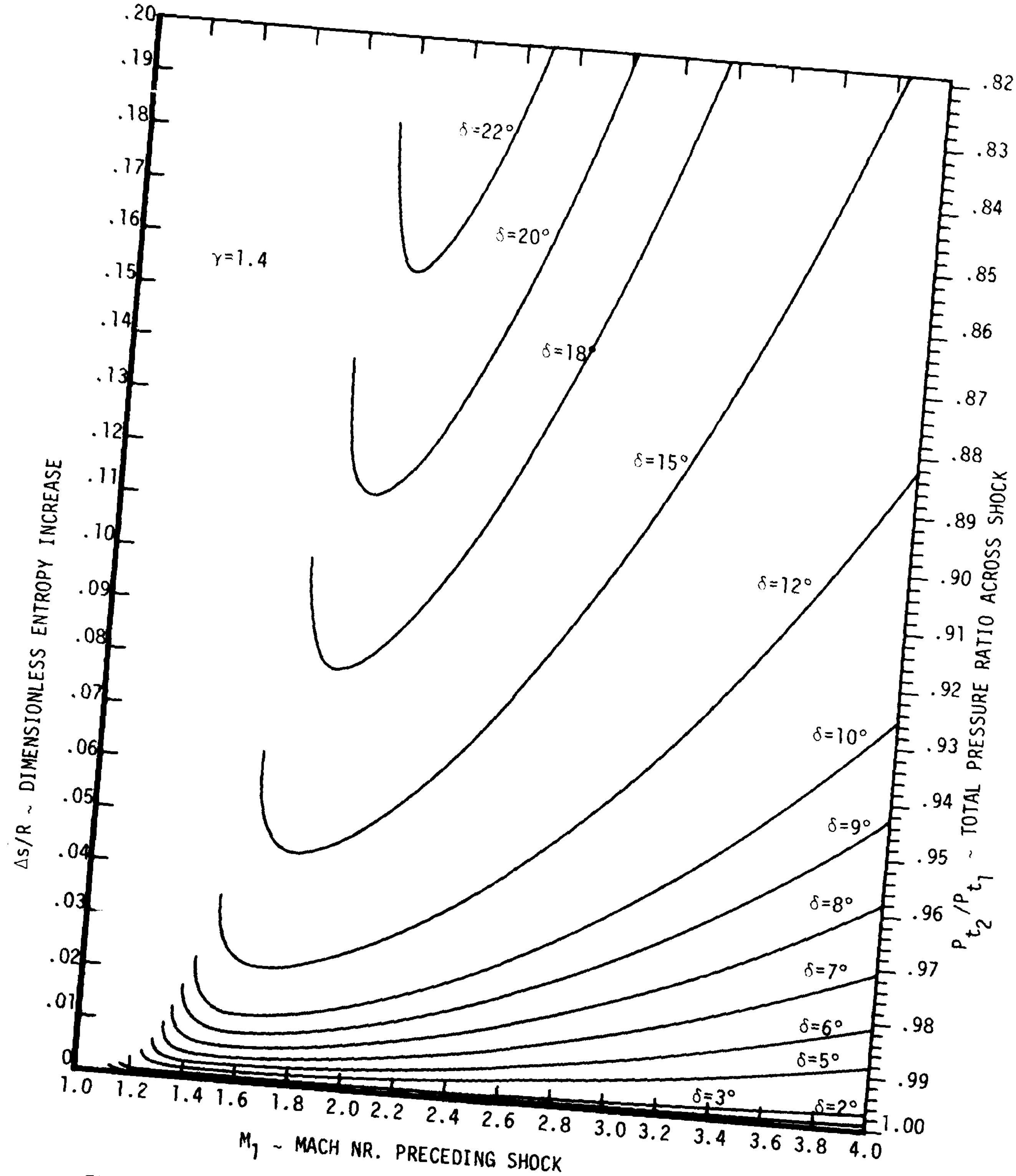


Figure B-7. Dimensionless Entropy Increase for Small 2-D Deflections

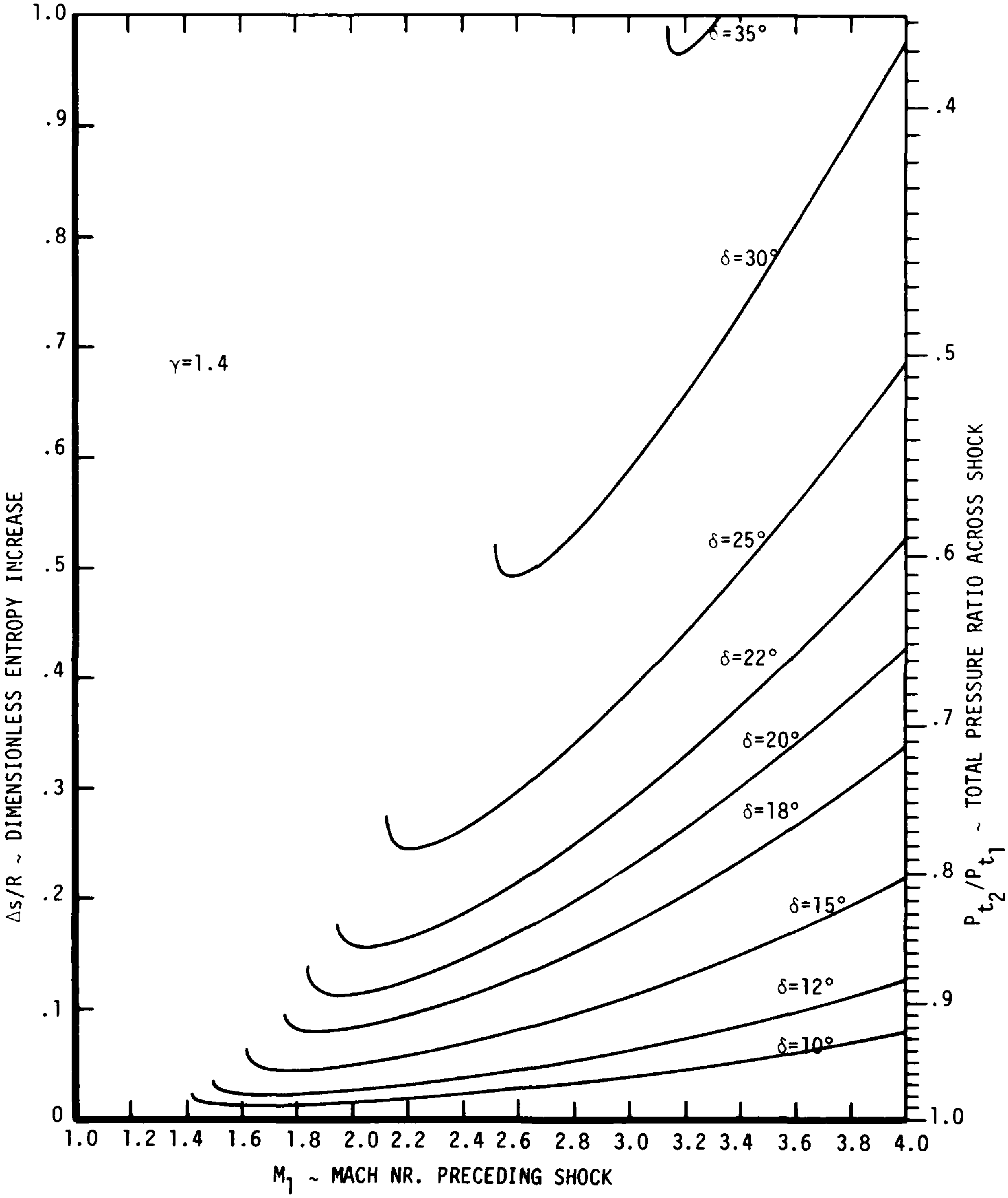


Figure B-8. Dimensionless Entropy Increase for Large 2-D Deflections

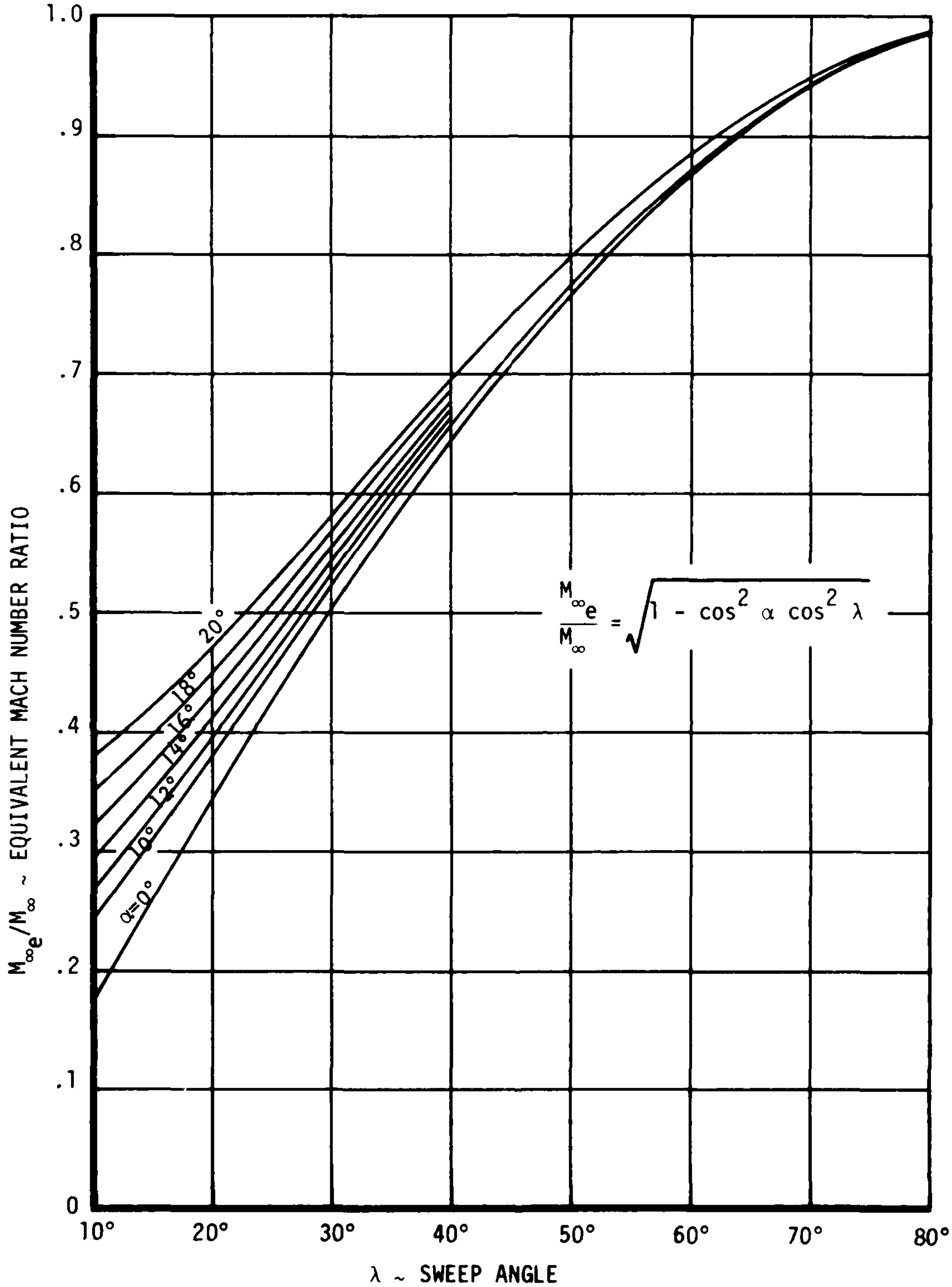


Figure B-9. Mach Number Conversion for Swept Wings

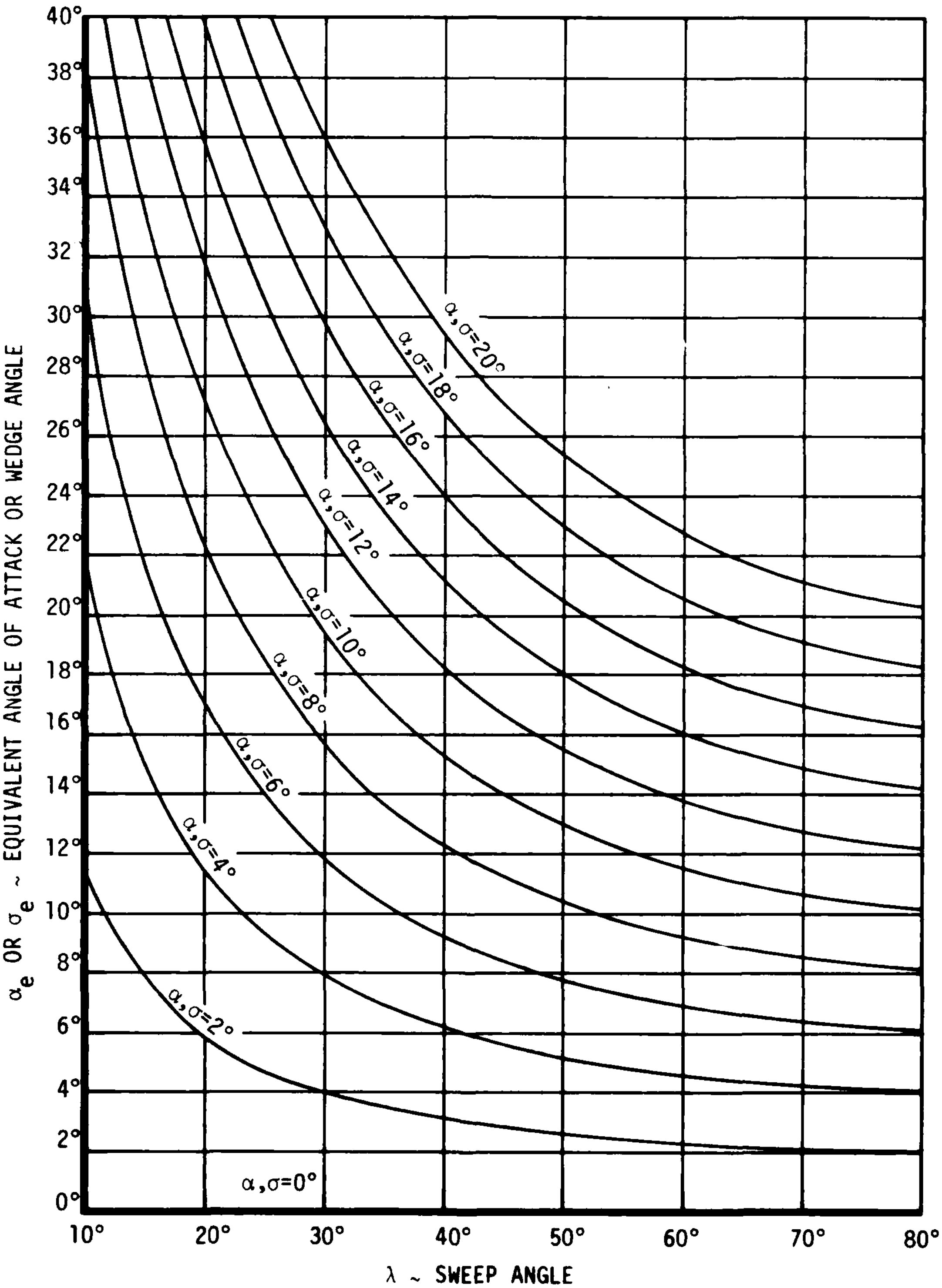
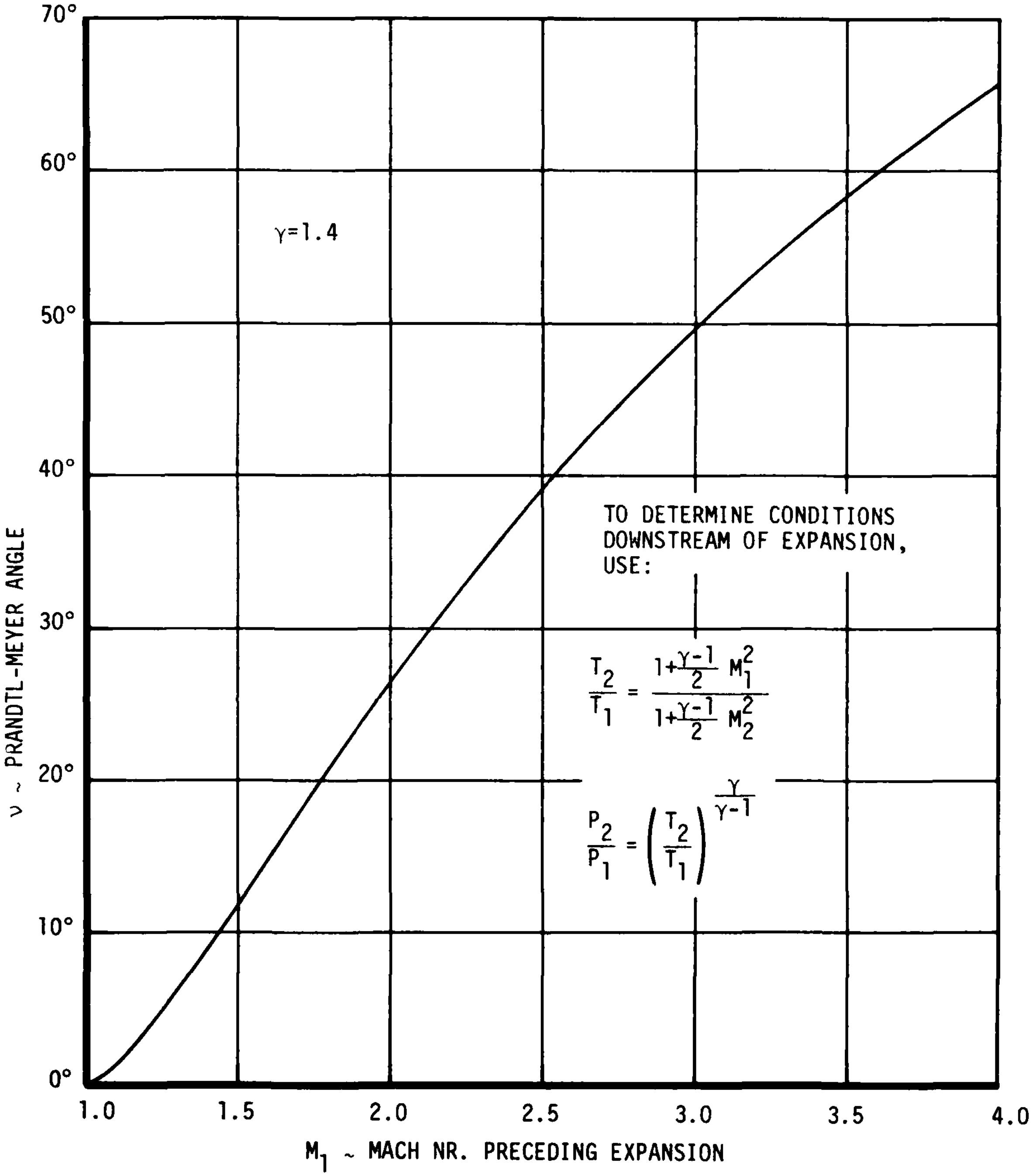


Figure B-10. Angular Conversions for Swept Wings



$$\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2} M_1^2}{1+\frac{\gamma-1}{2} M_2^2}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

Figure B-11. Prandtl-Meyer Angle

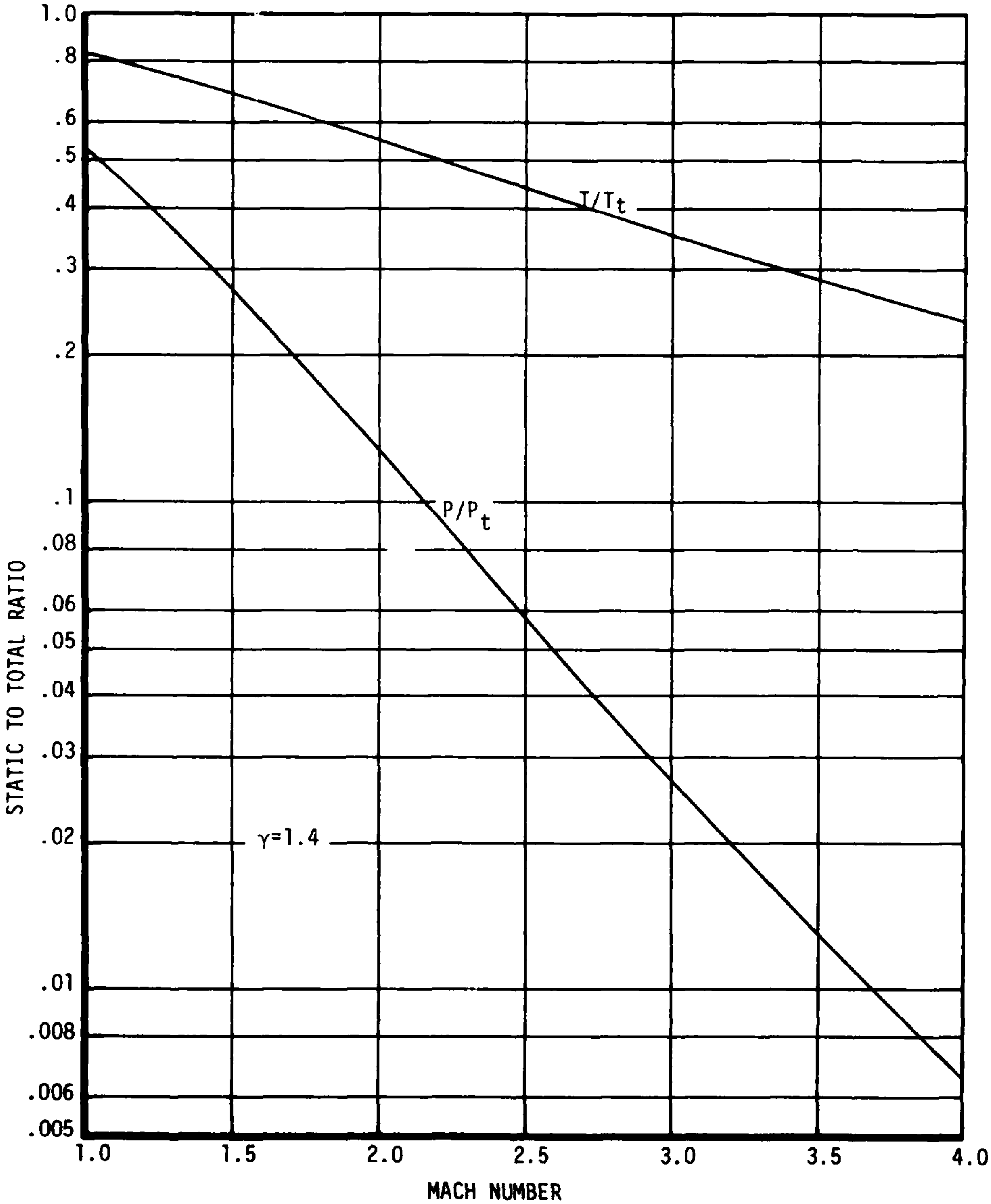


Figure B-12. Static to Total Ratios

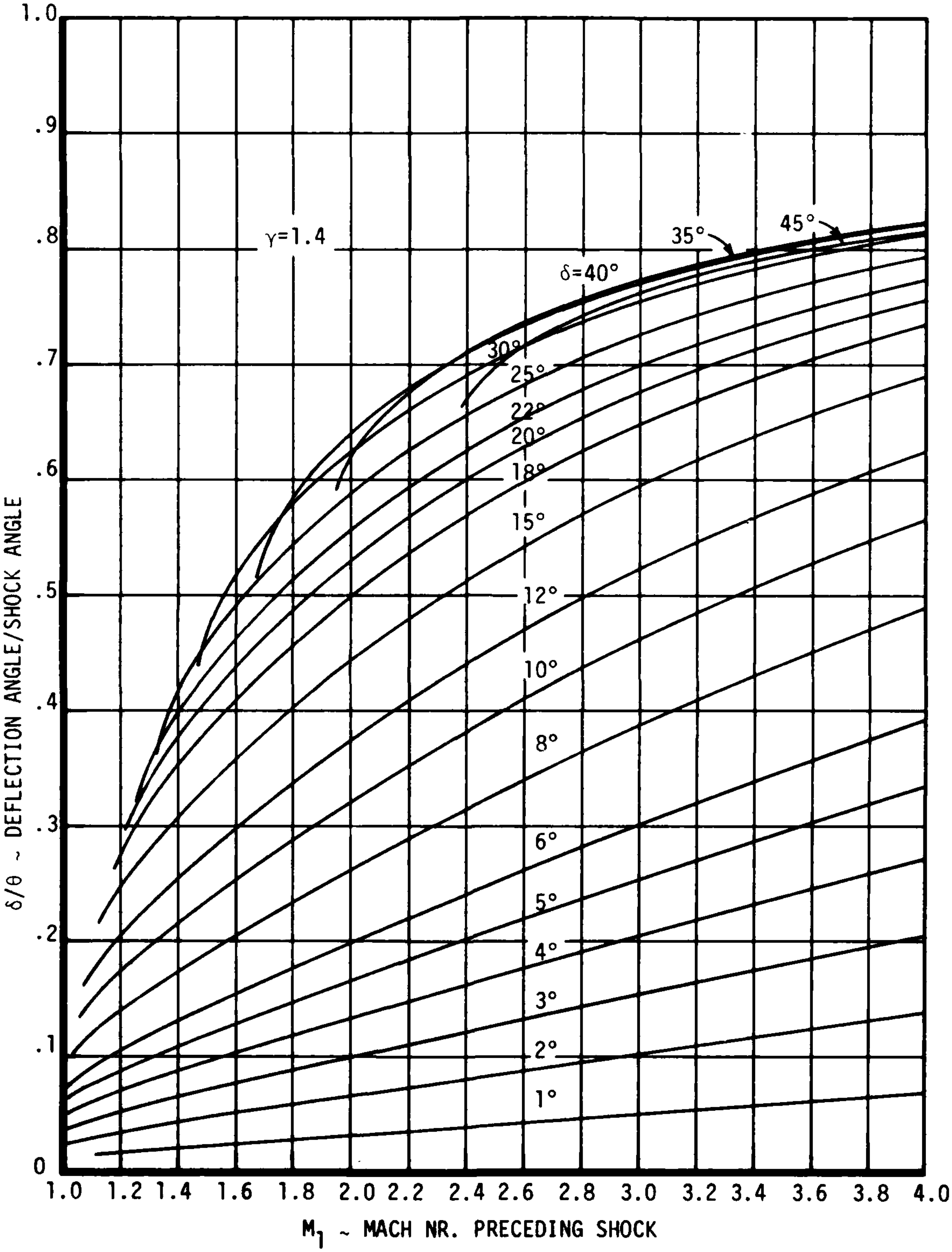


Figure B-13. Shock Wave Angle for Cones

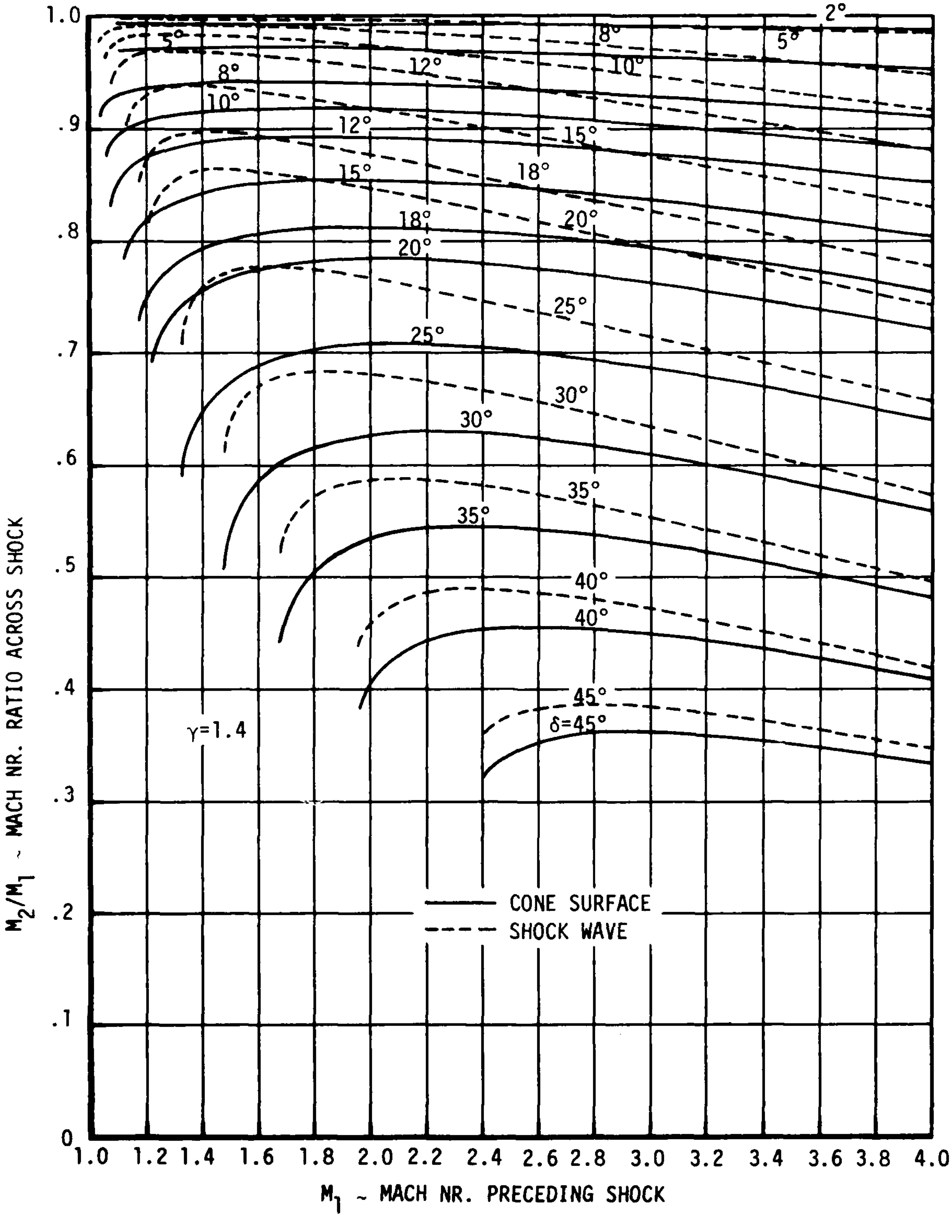


Figure B-14. Mach Number Ratio for Cones

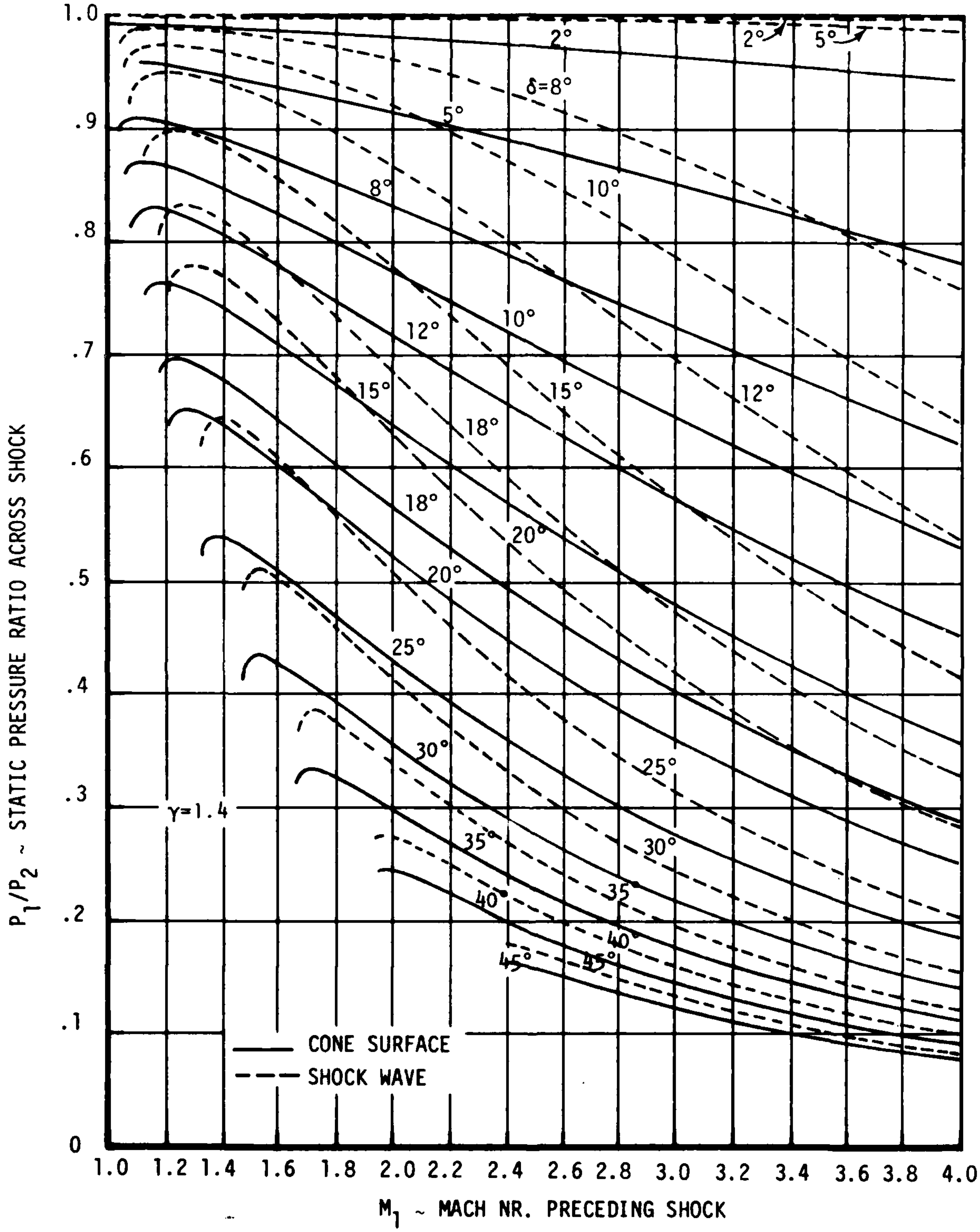


Figure B-15. Static Pressure Ratio for Cones

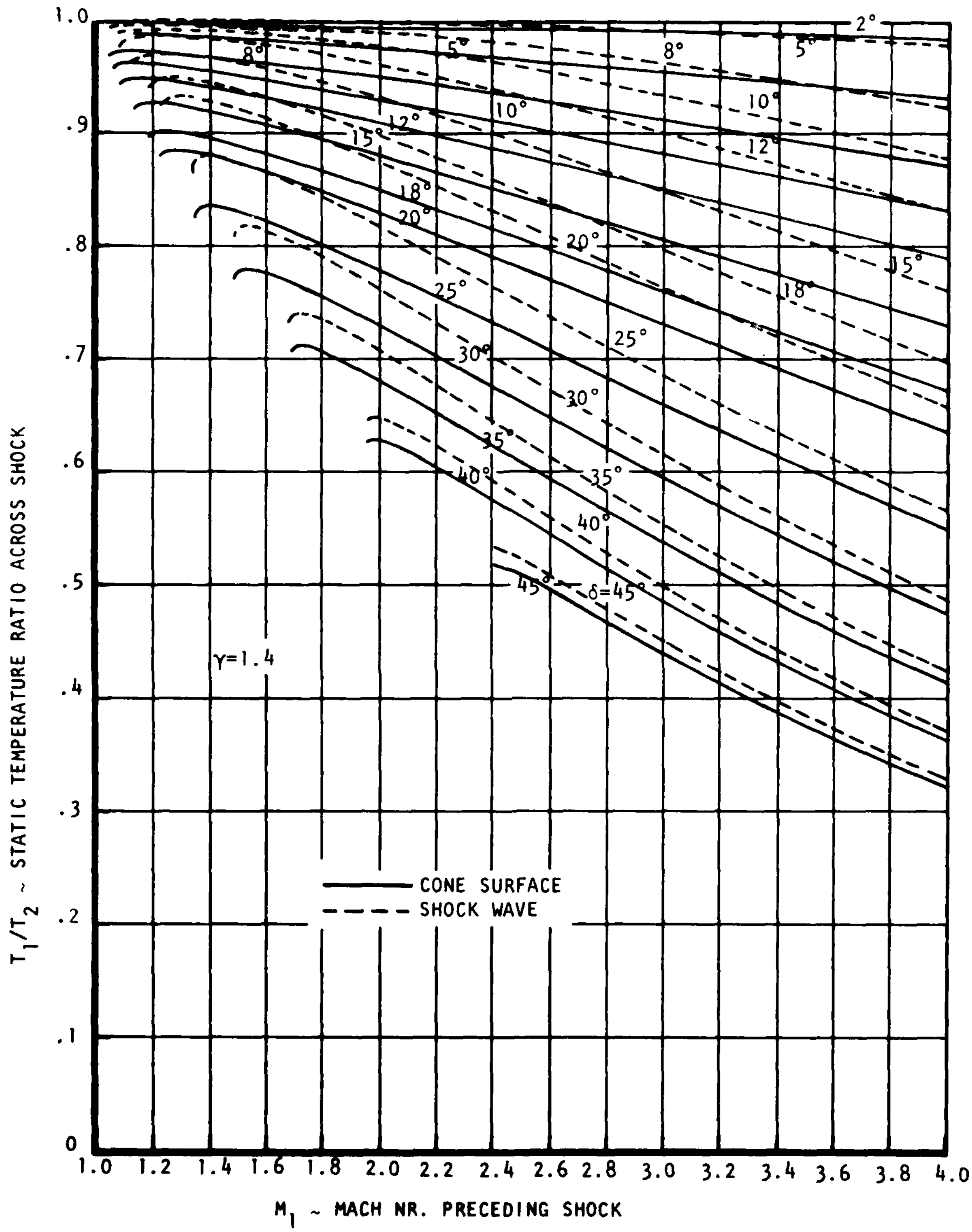


Figure B-16. Static Temperature Ratio for Cones

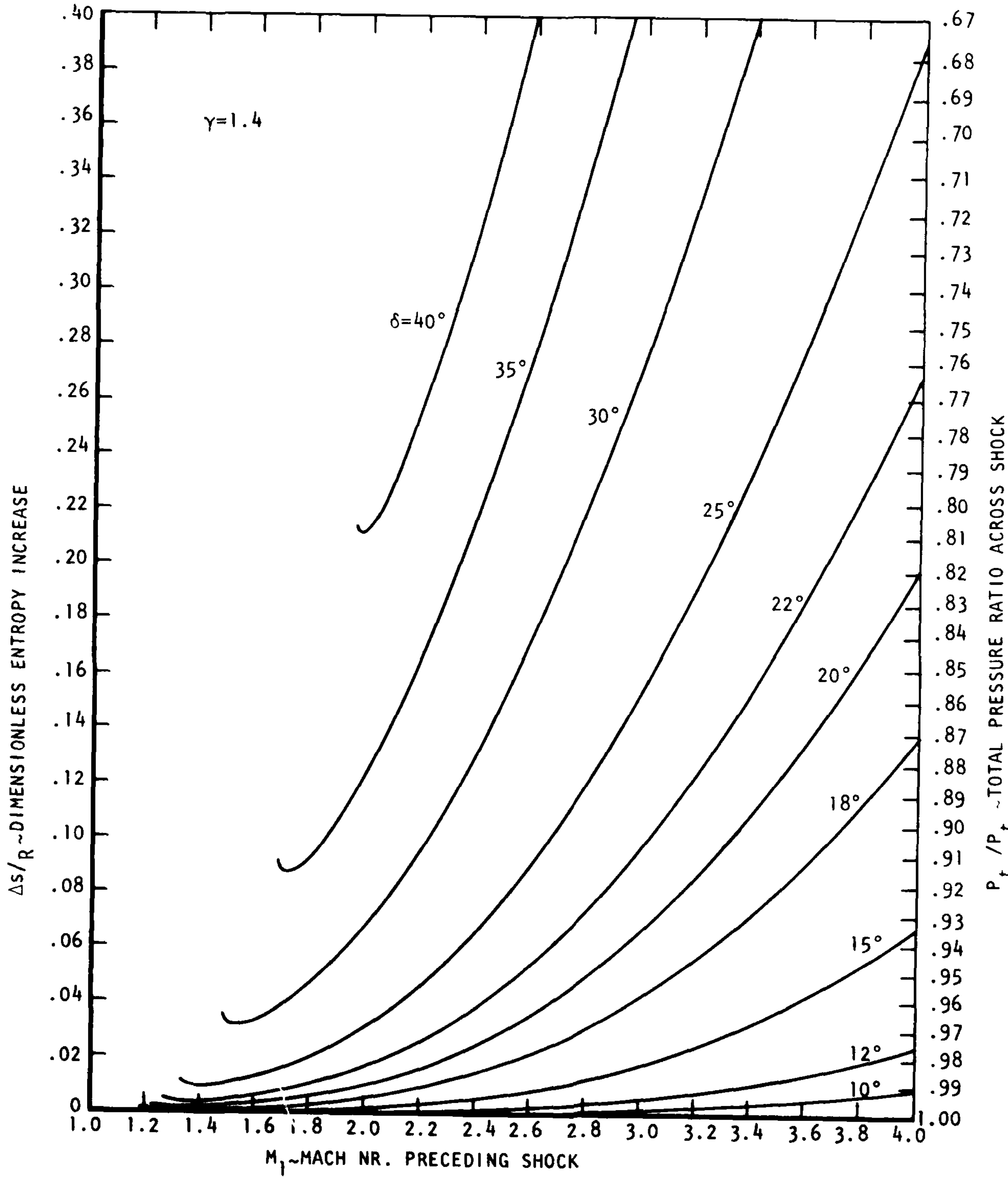


Figure B-17. Dimensionless Entropy Increase for Cones

REFERENCES

1. Hansen, C. Frederick, and Marion, E. Hodge, "Constant Entropy Properties for an Approximate Model of Equilibrium Air", NASA TN D-352, 1961.

2. Ames Research Staff, "Equations, Tables, and Charts for Compressible Flow", NACA Report 1135, 1953.

3. Shapiro, Ascher H., "The Dynamics and Thermodynamics of Compressible Fluid Flow", The Ronald Press Company, New York, 1953.

4. Dailey, C. L., and Wood, F. C., "Computation Curves for Compressible Fluid Problems", John Wiley & Sons, New York, 1949.

5. Moeckel, W. E., "Oblique-Shock Relations at Hypersonic Speeds for Air in Chemical Equilibrium", NACA TN 3895, 1957.

6. Moeckel, W. E., and Weston, Kenneth C., "Composition and Thermodynamic Properties of Air in Chemical Equilibrium", NACA TN 4265, 1958.

7. Eggers, A. J., Jr, and Syvertson, Clarence A., "Inviscid Flow About Airfoils at High Supersonic Speeds", NACA TN 2646, 1952.

8. Kopal, Zdenek, "Tables of Supersonic Flow of Air Around Cones", Massachusetts Institute of Technology Technical Report No. 1, 1947.

9. Kopal, Zdenek, "Tables of Supersonic Flow Around Yawing Cones", Massachusetts Institute of Technology Technical Report No. 3, 1947.

10. Kopal, Zdenek, "Tables of Supersonic Flow Around Cones of Large Yaw", Massachusetts Institute of Technology Technical Report No. 5, 1949.

11. Burington, R. S., "Handbook of Mathematical Tables and Formulas", Handbook Publishers, Inc., Sandusky, Ohio, 1957.

12. Moore, F. K., "Laminar Boundary Layer on a Circular Cone in Supersonic Flow at a Small Angle of Attack", NACA TN 2521, 1951.

13. Henderson, Arthur, Jr., "Investigation of the Flow Over Simple Bodies at Mach Numbers of the Order of 20", NASA TN D-449, 1960.

14. Huber, P. W., "Tables and Graphs of Normal-Shock Parameters at Hypersonic Mach Numbers and Selected Altitudes", NACA TN 4352, 1958.

15. Mirels, Harold, "Approximate Analytical Solutions for Hypersonic Flow Over Slender Power Law Bodies", NASA TR R-15, 1959.

16. Ferri, Antonio, "The Method of Characteristics for the Determination of Supersonic Flow Over Bodies of Revolution at Small Angles of Attack", NACA Report 1044, 1951.

REFERENCES (CONCLUDED)

17. Ferri, Antonio, "Supersonic Flow Around Circular Cones at Angles of Attack", NACA Report 1045, 1951.

18. Newman, P. A., "Approximate Calculation of Hypersonic Conical Flow Parameters for Air In Thermodynamics Equilibrium", NASA TN D-2058, 1964.

19. Bertram, M. H., "Correlation Graphs for Supersonic Flow Around Right Circular Cones at Zero Yaw In Air as a Perfect Gas", NASA TN D-2339,

20. Smith, G. L., Erickson, W. D., and Eastwood, M. R., "Equations for the Rapid Machine Computation of Equilibrium Composition of Air and Derivatives for Flow Field Calculations", NASA TN D-4103, 1967.

21. Anderson, B. H., "Design of Supersonic Inlets by a Computer Program Incorporating the Method of Characteristics", NASA TN D-4960, 1969.

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